

Series ABCD5/5

SET No. 3



प्रश्न पत्र कोड
Q.P. Code

65/5/3

रोल नं.

Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 7 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 14 प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 7 printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 14 questions.
- Please write down the Serial Number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.



गणित

MATHEMATICS



निर्धारित समय : 2 घण्टे
Time allowed : 2 hours

अधिकतम अंक : 40
Maximum Marks : 40

65/5/3

1

[P.T.O.]



सामान्य निर्देश:

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

1. इस प्रश्न-पत्र के तीन खण्ड- क, ख तथा ग हैं।
2. प्रत्येक खण्ड अनिवार्य हैं।
3. खण्ड-क में 6 लघु-उत्तर प्रकार-I के प्रश्न हैं जिनमें प्रत्येक के 2 अंक हैं।
4. खण्ड-ख में 4 लघु-उत्तर प्रकार-II के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।
5. खण्ड-ग में 4 दीर्घ-उत्तरीय प्रश्न हैं जिनमें प्रत्येक के 4 अंक हैं।
6. कुछ प्रश्नों में आंतरिक विकल्प दिया गया है।
7. प्रश्न 14 एक प्रकरण अध्ययन आधारित प्रश्न है जिसमें दो भाग हैं जिनमें से प्रत्येक के 2 अंक हैं।

खण्ड क

प्रश्न संख्या 1 से 6 तक प्रत्येक प्रश्न के 2 अंक हैं।

1. रेखा AB का कार्तीय समीकरण है : 2

$$\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3}$$

रेखा AB के समांतर रेखा के दिक्-कोसाइन ज्ञात कीजिए।

2. ज्ञात कीजिए : $\int \frac{dx}{\sqrt{4x-x^2}}$ 2

3. (क) घटनाओं A और B के लिए 2

$$P(A) = \frac{1}{2}, P(B) = \frac{7}{12} \text{ और } P(\bar{A} \cup \bar{B}) = \frac{1}{4} \text{ है।}$$

ज्ञात कीजिए कि क्या A और B स्वतंत्र घटनाएँ हैं या नहीं।

अथवा

- (ख) एक बॉक्स B_1 में 1 सफेद गेंद और 3 लाल गेंदें हैं। दूसरे बॉक्स B_2 में 2 सफेद गेंदें तथा 3 लाल गेंदें हैं। प्रत्येक बॉक्स B_1 तथा B_2 से एक-एक गेंद यादृच्छया निकाली जाती है। प्रायिकता ज्ञात कीजिए कि दोनों गेंदें एक ही रंग की हों। 2

4. निम्न अवकल समीकरण : 2

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

का व्यापक हल ज्ञात कीजिए।



General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This question paper contains **three Sections-A, B and C.**
2. **Each section is compulsory.**
3. **Section-A has 6 short-answer type-I questions of 2 marks each.**
4. **Section-B has 4 short-answer type-II questions of 3 marks each.**
5. **Section-C has 4 long-answer type questions of 4 marks each.**
6. **There is an internal choice in some questions.**
7. **Question 14 is a case study based question with two subparts of 2 marks each.**

SECTION A

Question numbers 1 to 6 carry 2 marks each.

1. The Cartesian equation of a line AB is : 2

$$\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3}$$

Find the direction cosines of a line parallel to line AB.

2. Find : $\int \frac{dx}{\sqrt{4x-x^2}}$ 2

3. (a) Events A and B are such that 2

$$P(A) = \frac{1}{2}, P(B) = \frac{7}{12} \text{ and } P(\bar{A} \cup \bar{B}) = \frac{1}{4}$$

Find whether the events A and B are independent or not.

OR

- (b) A box B_1 contains 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2 , then find the probability that the two balls drawn are of the same colour. 2

4. Find the general solution of the following differential equation : 2

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$



5. यदि $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ और $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$ हो, तो $|\vec{b}|$ ज्ञात कीजिए। 2

6. x_1, x_2, x_3, x_4 यादृच्छिक चर X के संभव मान (मूल्य) इस प्रकार हैं कि 2
 $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$,
चर X का प्रायिकता बंटन ज्ञात कीजिए।

खण्ड ख

प्रश्न संख्या 7 से 10 तक प्रत्येक प्रश्न के 3 अंक हैं।

7. (क) यदि दो सदिशों \vec{a} और \vec{b} के लिए $|\vec{a} + \vec{b}| = |\vec{b}|$ है, तो सिद्ध कीजिए कि सदिश 3
 $(\vec{a} + 2\vec{b})$, सदिश \vec{a} के लम्बवत् होगा।

अथवा

(ख) यदि \vec{a} और \vec{b} मात्रक सदिश हैं और उनके बीच का कोण θ है, तो सिद्ध कीजिए कि 3

$$\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}|$$

8. (क) ज्ञात कीजिए : 3
 $\int e^x \cdot \sin 2x \, dx$

अथवा

(ख) ज्ञात कीजिए : 3

$$\int \frac{2x}{(x^2+1)(x^2+2)} \, dx$$

9. मान ज्ञात कीजिए : $\int_1^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4-x}} \, dx$ 3

10. रेखा $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ के अनुदिश बिन्दु (2, 3, 4) की समतल $3x+2y+2z+5=0$ से दूरी 3
ज्ञात कीजिए।



5. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then find $|\vec{b}|$ 2

6. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$. Find the probability distribution of X . 2

SECTION B

Question numbers 7 to 10 carry 3 marks each.

7. (a) If \vec{a} and \vec{b} are two vectors such that $|\vec{a} + \vec{b}| = |\vec{b}|$, then prove that $(\vec{a} + 2\vec{b})$ is perpendicular to \vec{a} . 3

OR

(b) If \vec{a} and \vec{b} are unit vectors and θ is the angle between them, then prove that $\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}|$. 3

8. (a) Find : $\int e^x \cdot \sin 2x \, dx$ 3

OR

(b) Find : $\int \frac{2x}{(x^2+1)(x^2+2)} \, dx$ 3

9. Evaluate : $\int_1^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4-x}} \, dx$ 3

10. Find the distance of the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x+2y+2z+5 = 0$. 3



खण्ड ग

प्रश्न संख्या 11 से 14 तक प्रत्येक प्रश्न के 4 अंक हैं।

11. समाकलन के प्रयोग से, वक्रों $y = |x-1|$ तथा $y = 1$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए। 4

12. (क) निम्न अवकल समीकरण का हल ज्ञात कीजिए : 4

$$(y - \sin^2 x)dx + \tan x dy = 0$$

अथवा

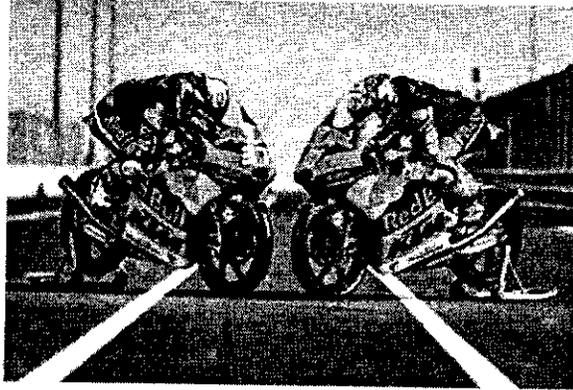
(ख) अवकल समीकरण $(x^3 + y^3)dy = x^2 y dx$ का व्यापक हल ज्ञात कीजिए। 4

13. दो बॉक्स, बॉक्स-I और बॉक्स-II दिए गए हैं। बॉक्स-I में 3 लाल और 6 काली गेंदें हैं। बॉक्स-II में 5 लाल और 5 काली गेंदें हैं। इन दो बॉक्सों में से एक बॉक्स यादृच्छया लिया जाता है और उससे एक गेंद यादृच्छया निकाली जाती है। निकाली गई गेंद लाल रंग की है। इस लाल रंग की गेंद के बॉक्स-II से आने की प्रायिकता ज्ञात कीजिए। 4

प्रकरण अध्ययन आधारित प्रश्न

14. दो मोटर साइकिलें A और B, अनुमत गति से अधिक गति से क्रमशः रेखाओं $2 \times 2 = 4$

$\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ और $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ द्वारा निरूपित सड़कों पर दौड़ रही हैं।



उपरोक्त पर आधारित होकर, निम्न प्रश्नों के उत्तर दीजिए :

(क) रेखाओं के बीच की न्यूनतम दूरी ज्ञात कीजिए। 2

(ख) वह बिन्दु ज्ञात कीजिए जहां पर यह दोनों मोटर साइकिलें आपस में टकरा सकती हैं। 2



SECTION C

Question numbers 11 to 14 carry 4 marks each.

11. Find the area bounded by the curves $y = |x-1|$ and $y = 1$, using integration. 4

12. (a) Solve the following differential equation : 4

$$(y - \sin^2 x)dx + \tan x dy = 0$$

OR

(b) Find the general solution of the differential equation : 4

$$(x^3 + y^3)dy = x^2 y dx$$

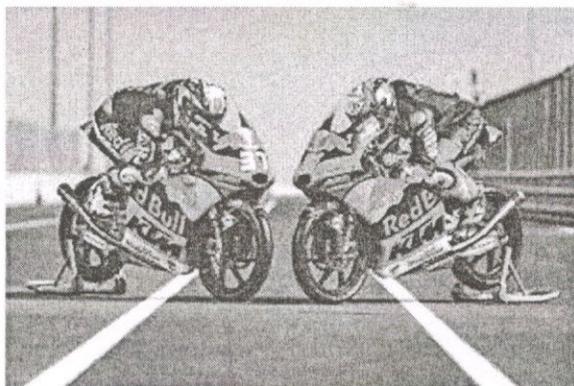
13. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes, is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II. 4

Case Study Based Question

14. Two motorcycles A and B are running at the speed more than the allowed speed $2 \times 2 = 4$

on the roads represented by the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and

$\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ respectively.



Based on the above information, answer the following questions :

(a) Find the shortest distance between the given lines. 2

(b) Find the point at which the motorcycles may collide. 2



SECTION C

Question numbers 11 to 14 carry 4 marks each.

11. Find the area bounded by the curve $y = x^2 - 4$ and $y = 1$, using integration.

(a) Solve the following differential equation:

$$(y - \sin x) dx + x dy = 0$$

OR

(b) Find the general solution of the differential equation:

$$(x^2 + y^2) dy = x y dx$$

12. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 2 red and 3 black balls. One of the two boxes is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II.

Case Study Based Question

13. Two motor cars A and B are running at the speed more than the allowed speed.

The roads represented by the lines $2x + 3y = 1$ and $x + 2y = 1$ and

$$x = 1 + 3t, y = 2t + 1$$
 respectively.



Based on the above information, answer the following questions:

- (a) Find the shortest distance between the given lines.
- (b) Find the point at which the motor cars may collide.

Strictly Confidential: (For Internal and Restricted use only)
Senior Secondary School Term II Examination, 2022
Marking Scheme – MATHEMATICS (SUBJECT CODE – 041)
(PAPER CODE – 65/1/1)

General Instructions: -

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2. **“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under IPC.”**
3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-XII, while evaluating two competency based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.**
4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
5. Evaluators will mark (√) wherever answer is correct. For wrong answer ‘X’ be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
10. A full scale of marks **0 TO 40** (example 0-40 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

11. Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 30 answer books per day in main subjects and 35 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
12. Ensure that you do not make the following common types of errors committed by the Examiner in the past: -
 - Leaving answer or part thereof unassessed in an answer book.
 - Giving more marks for an answer than assigned to it.
 - Wrong totaling of marks awarded on a reply.
 - Wrong transfer of marks from the inside pages of the answer book to the title page.
 - Wrong question wise totaling on the title page.
 - Wrong totaling of marks of the two columns on the title page.
 - Wrong grand total.
 - Marks in words and figures not tallying.
 - Wrong transfer of marks from the answer book to online award list.
 - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
 - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
14. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

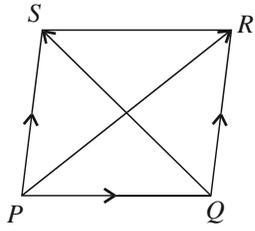
MARKING SCHEME

Senior Secondary School Examination TERM-II, 2022

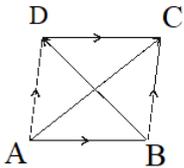
MATHEMATICS (Subject Code-041)

[Paper Code: 65/1/1]

Maximum Marks: 40

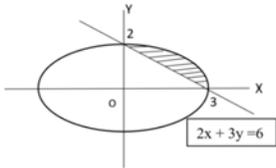
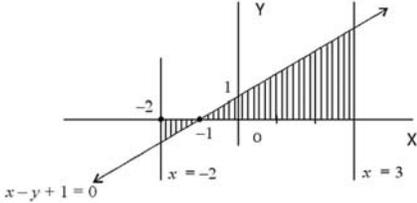
Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
SECTION-A		
1.	Find the sum of the order and the degree of the differential equation : $\left(x + \frac{dy}{dx}\right)^2 = \left(\frac{dy}{dx}\right)^2 + 1$	2
Sol.	Given differential equation can be written as $x^2 + \left(\frac{dy}{dx}\right)^2 + 2x \frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2 + 1$ i.e. $x^2 + 2x \frac{dy}{dx} = 1$; Order = 1, degree = 1 Sum of order and degree = 1 + 1 = 2	½ + ½ 1 2
2.	In a parallelogram PQRS, $\vec{PQ} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{PS} = -\hat{i} - 2\hat{k}$. Find $ \vec{PR} $ and $ \vec{QS} $.	2
Sol.	$\vec{PR} = \vec{PQ} + \vec{PS} \quad \therefore \vec{PS} = \vec{QR}$ $= 2\hat{i} - 2\hat{j}$ $ \vec{PR} = 2\sqrt{2}$ $\vec{QS} = \vec{QP} + \vec{PS} = \vec{PS} - \vec{PQ}$ $= -4\hat{i} + 2\hat{j} - 4\hat{k}$ $ \vec{QS} = 6$	 1 1 2
3.	(a) If $\frac{d}{dx} [F(x)] = \frac{\sec^4 x}{\operatorname{cosec}^4 x}$ and $F\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$, then find F(x). OR (b) Find : $\int \frac{\log x - 3}{(\log x)^4} dx$.	2
Sol.	$F(x) = \int \tan^4 x dx = \int \tan^2 x \cdot (\sec^2 x - 1) dx$	½

	$= \int (\tan^2 x \sec^2 x - \sec^2 x + 1) dx$ $F(x) = \frac{\tan^3 x}{3} - \tan x + x + C$ $x = \frac{\pi}{4}, F(x) = \frac{\pi}{4} \text{ gives } C = \frac{2}{3}$ $F(x) = \frac{\tan^3 x}{3} - \tan x + x + \frac{2}{3}$	1								
		1/2								
	Or									
	Put $x = e^t$ or $\log x = t$ $dx = e^t dt$	1/2								
	$\therefore \int \frac{\log x - 3}{(\log x)^4} dx = \int \frac{t-3}{t^4} e^t dt$ $= \int \left(\frac{1}{t^3} - \frac{3}{t^4} \right) e^t dt$ $= \frac{e^t}{t^3} + C$ $= \frac{x}{(\log x)^3} + C$	1/2								
		1/2								
		2								
4.	Let A and B be two events such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ and $P(A/B) = \frac{3}{4}$. Find the value of $P(B/A)$.	2								
Sol.	$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{3}{4} \Rightarrow P(A \cap B) = \frac{3}{8}$ $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{5}$	1								
		1								
		2								
5.	Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variable X denotes the number of red balls. Find the probability distribution of X.	2								
Sol.	<table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>P(X)</td> <td>$\frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$</td> <td>$2 \left(\frac{2}{5} \times \frac{3}{4} \right) = \frac{6}{10}$</td> <td>$\frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$</td> </tr> </tbody> </table>	X	0	1	2	P(X)	$\frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$	$2 \left(\frac{2}{5} \times \frac{3}{4} \right) = \frac{6}{10}$	$\frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$	1/2
X	0	1	2							
P(X)	$\frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$	$2 \left(\frac{2}{5} \times \frac{3}{4} \right) = \frac{6}{10}$	$\frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$							
		1/2								

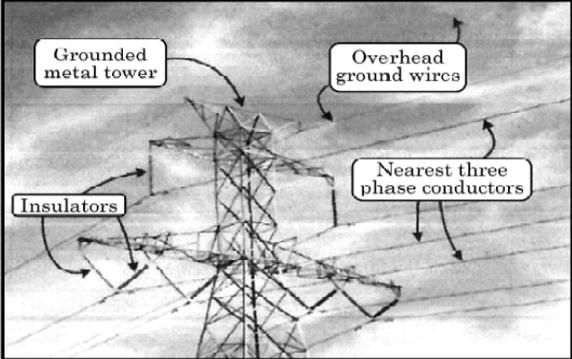
		2	
6.	Find the values of λ , for which the distance of point $(2, 1, \lambda)$ from plane $3x + 5y + 4z = 11$ is $2\sqrt{2}$ units.	2	
Sol.	$\frac{ 3 \cdot 2 + 5 \cdot 1 + 4\lambda - 11 }{\sqrt{9 + 25 + 16}} = 2\sqrt{2}$ $ 4\lambda = 20$ $\lambda = \pm 5$	1	
		1	
		2	
SECTION - B			
7.	<p>(a) If $\vec{a}, \vec{b}, \vec{c}$ and \vec{d} are four non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = 4\vec{b} \times \vec{d}$, then show that $(\vec{a} - 2\vec{d})$ is parallel to $(2\vec{b} - \vec{c})$ where $\vec{a} \neq 2\vec{d}, \vec{c} \neq 2\vec{b}$.</p> <p style="text-align: center;">OR</p> <p>(b) The two adjacent sides of a parallelogram are represented by $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.</p>	3	
Sol.	<p>(a) Consider $(\vec{a} - 2\vec{d}) \times (2\vec{b} - \vec{c})$</p> $= \vec{a} \times 2\vec{b} - \vec{a} \times \vec{c} - 4\vec{d} \times \vec{b} + 2\vec{d} \times \vec{c}$ $= 0$ $\therefore (\vec{a} - 2\vec{d}) \parallel (2\vec{b} - \vec{c})$	$\frac{1}{2}$ 1 1 $\frac{1}{2}$ 3	
(b)	<p style="text-align: center;">Or</p> <p>Let ABCD be a parallelogram with</p> $\vec{AB} = \vec{DC} = 2\hat{i} - 4\hat{j} - 5\hat{k}$ <p>and</p> $\vec{BC} = \vec{AD} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ $\vec{AC} = \vec{AB} + \vec{BC} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ <p>and $\vec{BD} = 6\hat{j} + 8\hat{k}$</p>		1
	$\therefore \vec{AC} = 2\sqrt{6} \text{ and } \vec{BD} = 10$ <p>\therefore Required unit vectors \hat{d}_1 and \hat{d}_2 are</p> $\hat{d}_1 = \frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} - \frac{\hat{k}}{\sqrt{6}} \text{ and } \hat{d}_2 = \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}$	1	

	<p>Now,</p> $\text{Area of } ABCD = \frac{1}{2} \vec{d}_1 \times \vec{d}_2 $ $= \frac{1}{2} \left \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & 6 & 8 \end{vmatrix} \right $ $= \frac{1}{2} -4\hat{i} - 32\hat{j} + 24\hat{k} $ $= \frac{1}{2} \sqrt{1616} = 2\sqrt{101}$	1
		3
8.	Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$ and $\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$ and through the point $(2, 1, 3)$.	3
Sol.	Equation of the plane is $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7 + \lambda [\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9] = 0$ or $\vec{r} \cdot [(2 + 2\lambda)\hat{i} + (2 + 5\lambda)\hat{j} + (-3 + 3\lambda)\hat{k}] - 7 - 9\lambda = 0$ $(2, 1, 3)$ lies on it $\therefore 2(2 + 2\lambda) + 1(2 + 5\lambda) + 3(-3 + 3\lambda) - 7 - 9\lambda = 0$ $9\lambda = 10 \Rightarrow \lambda = 10/9$ Equation is $\vec{r} \cdot (38\hat{i} + 68\hat{j} + 3\hat{k}) = 153$	1
		1½
		½
		3
9.	(a) Find : $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$ OR (b) Evaluate : $\int_0^{\pi/2} \frac{\cos x}{(1 + \sin x)(4 + \sin x)} dx$.	3
Sol.		
(a)	$I = \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$ $= \int \frac{6y^5 dy}{y^3 + y^2} \quad x = y^6 \text{ so that } dx = 6y^5 dy$ $= 6 \int \frac{y^3}{y+1} dy$ $= 6 \int \left[(y^2 - y + 1) - \frac{1}{y+1} \right] dy$	1
		1

(b)	$= 6 \left[\frac{y^3}{3} - \frac{y^2}{2} + y - \log y+1 \right] + C$ $= 2\sqrt{x} - 3\sqrt[3]{x} + 6x^{1/6} - 6\log(x^{1/6} + 1) + C$	1/2
	Or	
	Let $\sin x = t$, then $\cos x dx = dt$	1/2
	$\therefore \int_0^{\pi/2} \frac{\cos x}{(1 + \sin x)(4 + \sin x)} dx = \int_0^1 \frac{dt}{(1+t)(4+t)}$ $= \frac{1}{3} \left[\int_0^1 \frac{1}{1+t} dt - \int_0^1 \frac{1}{4+t} dt \right]$ $= \frac{1}{3} \left[\log(1+t) \Big _0^1 - \log(4+t) \Big _0^1 \right]$ $= \frac{1}{3} [\log 2 - \log 5 + \log 4]$ <p style="text-align: center;">or</p> $= \frac{1}{3} \log \frac{8}{5}$	1/2
		3
10.	Find the particular solution of the differential equation $x \frac{dy}{dx} + x \cos^2\left(\frac{y}{x}\right) = y$; given that when $x = 1$, $y = \frac{\pi}{4}$.	3
Sol.	Given differential equation can be written as $\frac{dy}{dx} = \frac{y}{x} - \cos^2 \frac{y}{x} \quad \dots (1)$ <p>Let $y = vx \therefore \frac{dy}{dx} = v + x \frac{dv}{dx}$</p> <p>(1) Becomes $v + x \frac{dv}{dx} = v - \cos^2 v$</p> $\sec^2 v dv = -\frac{dx}{x}$ <p>Integrating both sides we get</p> $\tan v = -\log x + c$ $\tan \frac{y}{x} = -\log x + c$ <p>$x = 1, y = \frac{\pi}{4} \Rightarrow c = 1$</p>	1/2
		1/2
		1

	\therefore Particular solution is $\tan \frac{y}{x} = -\log x + 1$	$\frac{1}{2}$
		3
	SECTION-C	
11.	(a) Using integration, find the area of the region $\{(x, y) : 4x^2 + 9y^2 \leq 36, 2x + 3y \geq 6\}$. OR (b) Using integration, find the area of the region bounded by lines $x - y + 1 = 0, x = -2, x = 3$ and x -axis.	4
Sol.	 <p>Correct figure</p> <p>Clearly points of intersection are (3, 0) and (0, 2)</p> <p>Required area $= \frac{2}{3} \int_0^3 \sqrt{9-x^2} dx - \frac{2}{3} \int_0^3 (3-x) dx$</p> $= \frac{2}{3} \left[\frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} + \frac{(3-x)^2}{2} \right]_0^3$ $= \frac{2}{3} \left[\frac{9\pi}{4} - \frac{9}{2} \right] \text{ or } \frac{3\pi}{2} - 3$	<p>1</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>4</p>
	Or	
	 <p>Correct figure</p> <p>Required area $= -\int_{-2}^{-1} (x+1) dx + \int_{-1}^3 (x+1) dx$</p>	<p>1</p> <p>1</p>

	$= -\frac{(x+1)^2}{2} \Big _{-2}^{-1} + \frac{(x+1)^2}{2} \Big _{-1}^3$ $= \frac{1}{2} + 8$ $= \frac{17}{2}$	1
		1
		4
12.	A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace.	4
Sol.	$\left. \begin{array}{l} E_1 : \text{Lost card is an ace} \\ E_2 : \text{Lost card is not an ace} \\ A : \text{2 ace cards are drawn} \end{array} \right\}$ $P(E_1) = \frac{1}{13} \qquad P(E_2) = \frac{12}{13}$ $P(A/E_1) = \frac{{}^3C_2}{{}^{51}C_2} \qquad P(A/E_2) = \frac{{}^4C_2}{{}^{51}C_2}$ $P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$ $= \frac{\frac{1}{13} \cdot \frac{{}^3C_2}{{}^{51}C_2}}{\frac{1}{13} \cdot \frac{{}^3C_2}{{}^{51}C_2} + \frac{12}{13} \cdot \frac{{}^4C_2}{{}^{51}C_2}} = \frac{3}{75} \text{ or } \frac{1}{25}$	1/2
		1
		1
		1 + 1/2
		4
13.	Evaluate : $\int_0^{\pi} \frac{x}{1 + \sin x} dx.$	4
Sol.	$I = \int_0^{\pi} \frac{x}{1 + \sin x} dx = \int_0^{\pi} \frac{\pi - x}{1 + \sin(\pi - x)} dx = \pi \int_0^{\pi} \frac{1}{1 + \sin x} dx - \int_0^{\pi} \frac{x}{1 + \sin x} dx$ $\therefore 2I = \pi \int_0^{\pi} \frac{dx}{1 + \sin x}$ $\therefore I = \frac{\pi}{2} \int_0^{\pi} \frac{dx}{1 + \sin x} = \frac{\pi}{2} \int_0^{\pi} \frac{1}{1 + \cos\left(\frac{\pi}{2} - x\right)} dx$	1
		1/2
		1/2

	$= \frac{\pi}{2} \int_0^{\pi} \frac{1}{2 \cos^2 \left(\frac{\pi - x}{4} \right)} dx$ $= \frac{\pi}{4} \left[-2 \tan \left(\frac{\pi - x}{4} \right) \right]_0^{\pi}$ $= \frac{\pi}{4} [2 - (-2)] = \pi$	<p>1</p> <p>1</p> <p>4</p>
<p>14.</p>	<p>Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.</p>  <p>Two such wires lie along the following lines :</p> $l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$ $l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$ <p>Based on the given information, answer the following questions :</p> <p>(i) Are the lines l_1 and l_2 coplanar ? Justify your answer.</p> <p>(ii) Find the point of intersection of the lines l_1 and l_2.</p>	<p>2</p> <p>2</p>
<p>Sol.</p>	<p>(i) Consider</p> $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$ $= \begin{vmatrix} +1 & 4 & -5 \\ 3 & -2 & -1 \\ -1 & 3 & -2 \end{vmatrix}$ $= 1(+7) - 4(-7) - 5(7)$ $= 0$ <p>\therefore lines l_1 and l_2 are coplanar</p> <p>(ii) Any point on l_1 : $(3\lambda - 1, -2\lambda + 3, -\lambda - 2)$</p> <p>Substituting in equation of l_2,</p>	<p>1</p> <p>1</p> <p>1</p>

	$\frac{3\lambda - 1}{-1} = \frac{-2\lambda + 3 - 7}{3}$ $\Rightarrow 9\lambda - 3 = 2\lambda + 4$ $\Rightarrow 7\lambda = 7 \Rightarrow \lambda = 1$ <p>Point is (2, 1, -3)</p>	$\frac{1}{2}$ $\frac{1}{2}$
		2 + 2