

KENDRIYA VIDYALAYA SANGATHAN LUCKNOW REGION

SECOND PREBOARD EXAM 2022-23

CLASS – XII

MATHEMATICS (041)

Time Allowed : 3 Hours

Maximum Marks : 80

**General Instructions:**

- 1 This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- 2 Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
- 3 Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
- 4 Section C has 6 Short Answer (SA)-type questions of 3 marks each.
- 5 Section D has 4 Long Answer (LA)-type questions of 5 marks each.
- 6 Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

**SECTION A**

**(Multiple Choice Questions) Each question carries 1 mark**

- 1 The corner points of the feasible region determined by the system of linear constraints are  $(0, 10)$ ,  $(5, 5)$ ,  $(15, 15)$ ,  $(0, 20)$ . Let  $Z = px + qy$ , where  $p, q > 0$ . Condition on  $p$  and  $q$  so that the maximum of  $Z$  occurs at both the points  $(15, 15)$  and  $(0, 20)$  is

- (a)  $p = q$                       (b)  $p = 2q$                       (c)  $q = 2p$                       (d)  $q = 3p$

- 2 If  $A$  is a square matrix such that  $A^2 = A$ , then the value of  $7A - (I + A)^3$  is, where  $I$  is an identity matrix

- (a)  $-I$                               (b)  $I$                               (c)  $A$                               (d)  $A - I$

- 3 If  $A_{ij}$  is the cofactor of the elements  $a_{ij}$  of the determinant  $|2 \ - \ 3 \ 5 \ 6 \ 0 \ 4 \ 1 \ 5 \ - \ 7|$  then the value of  $a_{32} \cdot A_{32}$  is

- (a)  $-110$                       (b)  $0$                               (c)  $-22$                               (d)  $110$

- 4 If  $f(x)$  is continuous at  $x = 3$  and  $f(x) = \left\{ \frac{(x+3)^2 - 36}{x-3}, x \neq 3 \right.$   $k, x = 3$  then the value of  $k$  is

- (a)  $3$                               (b)  $12$                               (c)  $9$                               (d)  $6$

- 5 The solution set of the inequality  $3x + 5y < 4$  is

- (a) an open half-plane not containing the origin.  
(b) an open half-plane containing the origin.

(c) the whole  $XY$ -plane not containing the line  $3x + 5y = 4$ .

(d) a closed half plane containing the origin.

6 Given  $\int e^x (\tan x + 1) \sec x dx = e^x f(x) + c$ , then  $f(x)$  is

- (a)  $\tan x$  (b)  $\sec x$  (c)  $\sec x \tan x$  (d) none of these.

7 If  $m$  and  $n$ , respectively, are the order and the degree of the differential equation  $\frac{d}{dx} \left[ \left( \frac{dy}{dx} \right) \right]^4 = 0$ , then  $m + n =$

- (a) 1 (b) 2 (c) 3 (d) 4

8 The general solution of the differential equation  $y dx - x dy = 0$  is

- (a)  $y = cx$  (b)  $xy = c$  (c)  $x = cy^2$  (d)  $y = cx^2$

9 If  $f(x) = \int_0^x y \sin y dy$ , then  $f'(x)$  is

- (a)  $\cos x + x \sin x$  (b)  $x \sin x$  (c)  $x \cos x$  (d)  $\sin x + x \cos x$

10 The scalar projection of the vector  $3\hat{i} - \hat{j} - 2\hat{k}$  on the vector  $\hat{i} + 2\hat{j} - 3\hat{k}$  is

- (a)  $\frac{7}{14}$  (b)  $\frac{7}{\sqrt{14}}$  (c) 7 (d)  $\frac{7}{2}$

11 Given a square matrix  $A$  of order  $3 \times 3$ , such that  $|A| = 12$ , then the value of  $|A \cdot \text{adj} A|$  is

- (a) 1728 (b) 144 (c) 12 (d) none of these

12 If  $|-1 \ 2 \ 4 \ 8| = |2 \ x \ x \ -4|$ ,  $x = ?$

- (a)  $\sqrt{2}$  (b)  $\sqrt{4}$  (c)  $2\sqrt{2}$  (d)  $\pm 2\sqrt{2}$

13 The value of  $x + y$  from the equation  $2(1 \ 3 \ 0 \ x) + (y \ 0 \ 1 \ 2) = (5 \ 6 \ 1 \ 8)$  is

- (a) 3 (b) 6 (c) 9 (d) 5

14. If  $\vec{a}$  and  $\vec{b}$  are such that  $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ , then the angle between  $\vec{a}$  and  $\vec{b}$  is

- (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{3}$  (c)  $\frac{\pi}{4}$  (d)  $\frac{\pi}{2}$

15 The value of  $i \cdot (j \times k) + j \cdot (k \times i) + k \cdot (j \times i)$  is

- (a) 1 (b) 3 (c) 0 (d) -1

16 If  $A$  and  $B$  are two independent events such that  $P(A) = \frac{1}{2}$ ,  $P(B) = p$  and  $P(A \cup B) = \frac{3}{5}$ , then the value of  $p$  is

- (a)  $\frac{1}{5}$  (b)  $\frac{2}{5}$  (c)  $\frac{3}{5}$  (d)  $\frac{4}{5}$

17 In a triangle  $OAC$ , if  $B$  is the midpoint of side  $AC$  and  $\vec{OA} = \vec{a}$ ,  $\vec{OB} = \vec{b}$ , then  $\vec{OC}$  is

- (a)  $2\vec{b}$       (b)  $2\vec{b} - \vec{a}$       (c)  $\frac{\vec{a} + \vec{b}}{2}$       (d)  $\frac{2\vec{b} - \vec{a}}{2}$

18. If  $x = t^2$  and  $y = t^3$ , then  $\frac{dy}{dx}$  is equal to

- (a)  $\frac{2t}{3}$       (b)  $2t$       (c)  $3t$       (d)  $\frac{3t}{2}$

### ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both  $A$  and  $R$  are true and  $R$  is the correct explanation of  $A$ .  
(b) Both  $A$  and  $R$  are true but  $R$  is not the correct explanation of  $A$ .  
(c)  $A$  is true but  $R$  is false.  
(d)  $A$  is false but  $R$  is true.

19 Assertion (A) :  $f: N \rightarrow N$  given by  $f(x) = 5x$  is injective but not surjective

Reason (R) : If co-domain  $\neq$  range, then the function is not surjective.

20 Assertion (A) : The direction-cosines of the line joining the points  $(1, 0, 0)$  and  $(0, 1, 1)$  is  $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Reason (R) : direction ratios = direction cosines.

### SECTION B

**This section comprises of very short answer type-questions (VSA) of 2 marks each**

21. Find the principal value of  $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ .

(OR)

Find the value of  $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]$

22. If  $\vec{a} = 3i + 2j + 9k$  and  $\vec{b} = i + \mu j + 3k$ , find  $\mu$ , so that  $\vec{a} + \vec{b}$  is perpendicular to  $\vec{a} - \vec{b}$

23. Write the vector of magnitude 15 units in the direction of  $i - 2j + 2k$ .

24 Find  $f'(x)$  if  $f(x) = (\sin x)^{\sin x}$ , for all  $0 < x < \pi$

25 The total revenue in Rupees received from the sale of  $x$  units of a product is given by

$$R(x) = 13x^2 + 26x + 15. \text{ Find the marginal revenue when } x = 7.$$

### SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26 Find  $\int \frac{1}{\sqrt{3-2x-x^2}} dx$

27 Solve the following linear programming problem graphically. Minimize  $Z = 3x + 9y$  subject to the constraints

$$x + 3y \leq 60, x + y \geq 10, x \leq y, x, y \geq 0$$

28. Evaluate:  $\int \frac{x^2+1}{x^2-5x+6} dx$

29. Evaluate:  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$

30. Solve:  $(x \sin \frac{y}{x}) dy = (y \sin \frac{y}{x} - x) dx$

(OR)

Solve:  $y dx + (x - y^2) dy = 0.$

31. A couple has two children. Find the probability that both the children are girls, if it is known that the older child is a girl.

(OR)

A random variable  $X$  has the following probability distribution, determine  $k$ .

$X$	0	1	2	3	4	5	6	7
$P(X)$	0	$k$	$2k$	$2k$	$3k$	$k^2$	$2k^2$	$7k^2 + k$

### SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32 Using matrix method, solve the following system of equations.

$$8x - 4y + z = 5$$

$$10x + 6z = 4$$

$$8x + y + 6z = \frac{5}{2}$$

(OR)

If  $A = \begin{pmatrix} 1 & -1 & 0 & 2 & 3 & 4 & 0 & 1 & 2 \end{pmatrix}$  and  $B = \begin{pmatrix} 2 & 2 & -4 & -4 & 2 & -4 & 2 & -1 & 5 \end{pmatrix}$  are two square matrices, find  $AB$  and hence solve the system of equations  $x - y = 3$ ,  $2x + 3y + 4z = 17$  and  $y + 2z = 7$

33. Find the shortest distance between the lines whose equations are

$$\vec{r}_1 = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k}) \text{ and } \vec{r}_2 = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$

(OR)

Find the vector equation of the line passing through the point  $(1, 2, -4)$  and

perpendicular to the two lines  $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$  and  $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

- 34 Show that the relation  $S$  in the set  $A = \{x \in Z: 0 \leq x \leq 12\}$  given by  $S = \{(a, b): a, b \in Z, |a - b| \text{ is divisible by } 4\}$  is an equivalence relation. Find the set of all elements related to 1 .
- 35 Using integration, find the area of the region in the first quadrant enclosed by the  $x$ -axis, the line  $y = x$  and the circle  $x^2 + y^2 = 32$ .

## SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each)

### 36. Case study 1:

Read the following information and answer the questions given below.



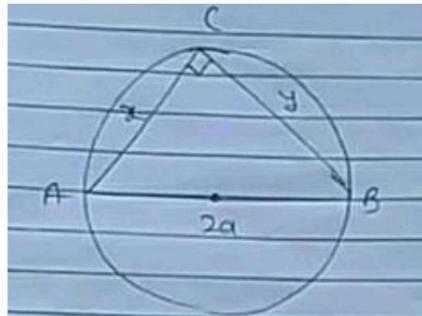
The temperature of a person during an intestinal illness is given by  $f(x) = -0.1x^2 + 1.2x + 98.6$ ,  $0 \leq x \leq 12$ , where  $f(x)$  is the temperature in  $^{\circ}F$  at  $x$  days.

- (i) Is the function differentiable in  $(0, 12)$  ? Justify your answer. (1)
- (ii) Find the critical point of the function in  $(0, 12)$  (1)

(iii) Find the intervals in which the function  $f(x)$  is strictly increasing or strictly decreasing. (2)

**37. Case study 2:**

In a circular field the authority wants to design a triangular field with the maximum possible area. The field is given by the following diagram with  $AB$  as a diameter of a circle and  $C$  is any point on the circle. Let  $AC = x$  and  $CB = y$



(a) Write  $y$  in terms of  $x$

(b) Write the Area of triangle in terms of  $x$

(c) Find the rate of change of area with respect to  $x$ .

(d) Find the value of  $x$  and  $y$ , when the area is maximum.

(4×1 = 4)

**38 Case study 3:**

In answering a question on a multiple-choice test for class XII, a student either knows the answer or guesses. Let  $3/5$  be the probability that he knows the answer and  $2/5$  be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability  $1/3$ . Let  $E_1, E_2, E$  be the events that the student knows the answer, guesses the answer and answers correctly respectively.



Based on the above information, answer the following

1 What is the value of  $P(E_1)$  ?

(1)

2 Find the value of  $P(E|E_1)$ .

(1)

3 What is the probability that the student knows the answer given that he answered it correctly? (2)