



Series EF1GH/4



SET~2

रोल नं.							
Roll No.							

प्रश्न-पत्र कोड
Q.P. Code **65/4/2**

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

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निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

नोट / NOTE :

- (i) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
Please check that this question paper contains 23 printed pages.
- (ii) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- (iii) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
Please check that this question paper contains 38 questions.
- (iv) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।
Please write down the serial number of the question in the answer-book before attempting it.
- (v) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.





सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) इस प्रश्न-पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- (ii) यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – क, ख, ग, घ एवं ङ।
- (iii) खण्ड क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित एक-एक अंक के प्रश्न हैं।
- (iv) खण्ड ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के दो-दो अंकों के प्रश्न हैं।
- (v) खण्ड ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के तीन-तीन अंकों के प्रश्न हैं।
- (vi) खण्ड घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के पाँच-पाँच अंकों के प्रश्न हैं।
- (vii) खण्ड ङ में प्रश्न संख्या 36 से 38 प्रकरण अध्ययन आधारित चार-चार अंकों के प्रश्न हैं।
- (viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
- (ix) कैल्कुलेटर का उपयोग वर्जित है।

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

1. यदि $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$ है, तो $(2x + y - z)$ का मान है :

- (a) 1 (b) 2
(c) 3 (d) 5

2. यदि आव्यूह $A = [1 \ 2 \ 3]$ है, तो आव्यूह AA' है (जहाँ A' आव्यूह A का परिवर्त है) :

(a) 14 (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (d) [14]





General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (ix) Use of calculators is **not** allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$, then the value of $(2x + y - z)$ is :

- (a) 1
- (b) 2
- (c) 3
- (d) 5

2. If a matrix $A = [1 \ 2 \ 3]$, then the matrix AA' (where A' is the transpose of A) is :

- (a) 14
- (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

- (c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$
- (d) $[14]$





3. यदि $x \begin{bmatrix} 1 \\ 2 \end{bmatrix} + y \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$ है, तो :

(a) $x = 1, y = 2$

(b) $x = 2, y = 1$

(c) $x = 1, y = -1$

(d) $x = 3, y = 2$

4. यदि A एक वर्ग आव्यूह है और $A^2 = A$ है, तो $(I + A)^2 - 3A$ बराबर है :

(a) I

(b) A

(c) 2A

(d) 3I

5. सारणिक $\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$ का मान है :

(a) 47

(b) -79

(c) 49

(d) -51

6. फलन $f(x) = |x|$

(a) हर जगह संतत व अवकलनीय है ।

(b) कहीं भी संतत व अवकलनीय नहीं है ।

(c) हर जगह संतत है, परन्तु $x = 0$ को छोड़कर शेष हर जगह अवकलनीय है ।

(d) हर जगह संतत है, परन्तु अवकलनीय कहीं भी नहीं है ।

7. यदि $y = \log(\sin e^x)$ है, तो $\frac{dy}{dx}$ है :

(a) $\cot e^x$

(b) $\operatorname{cosec} e^x$

(c) $e^x \cot e^x$

(d) $e^x \operatorname{cosec} e^x$

8. $\int e^{5 \log x} dx$ बराबर है :

(a) $\frac{x^5}{5} + C$

(b) $\frac{x^6}{6} + C$

(c) $5x^4 + C$

(d) $6x^5 + C$





3. If $x \begin{bmatrix} 1 \\ 2 \end{bmatrix} + y \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$, then :

- (a) $x = 1, y = 2$ (b) $x = 2, y = 1$
(c) $x = 1, y = -1$ (d) $x = 3, y = 2$

4. If A is a square matrix and $A^2 = A$, then $(I + A)^2 - 3A$ is equal to :

- (a) I (b) A
(c) 2A (d) 3I

5. The value of the determinant $\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$ is :

- (a) 47 (b) -79
(c) 49 (d) -51

6. The function $f(x) = |x|$ is

- (a) continuous and differentiable everywhere.
(b) continuous and differentiable nowhere.
(c) continuous everywhere, but differentiable everywhere except at $x = 0$.
(d) continuous everywhere, but differentiable nowhere.

7. If $y = \log(\sin e^x)$, then $\frac{dy}{dx}$ is :

- (a) $\cot e^x$ (b) $\operatorname{cosec} e^x$
(c) $e^x \cot e^x$ (d) $e^x \operatorname{cosec} e^x$

8. $\int e^{5 \log x} dx$ is equal to :

- (a) $\frac{x^5}{5} + C$ (b) $\frac{x^6}{6} + C$
(c) $5x^4 + C$ (d) $6x^5 + C$





9. $\int_0^4 (e^{2x} + x) dx$ बराबर है :

(a) $\frac{15 + e^8}{2}$

(b) $\frac{16 - e^8}{2}$

(c) $\frac{e^8 - 15}{2}$

(d) $\frac{-e^8 - 15}{2}$

10. सदिश $4\hat{i} - 3\hat{k}$ की दिशा में एक मात्रक सदिश है :

(a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$

(b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$

(c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$

(d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$

11. यदि दो सदिशों \vec{a} तथा \vec{b} के बीच का कोण θ है, तो $\vec{a} \cdot \vec{b} \geq 0$ केवल तब होगा, जब :

(a) $0 < \theta < \frac{\pi}{2}$

(b) $0 \leq \theta \leq \frac{\pi}{2}$

(c) $0 < \theta < \pi$

(d) $0 \leq \theta \leq \pi$

12. अवकल समीकरण $x \frac{dy}{dx} - y = 2x^2$ को हल करने के लिए समाकलन गुणक है :

(a) e^{-y}

(b) e^{-x}

(c) x

(d) $\frac{1}{x}$

13. अवकल समीकरण $\frac{dy}{dx} = \frac{y+1}{x-1}$, जब $y(1) = 2$ के हलों की संख्या है :

(a) शून्य

(b) एक

(c) दो

(d) अनन्त





9. $\int_0^4 (e^{2x} + x) dx$ is equal to :

- (a) $\frac{15 + e^8}{2}$ (b) $\frac{16 - e^8}{2}$
(c) $\frac{e^8 - 15}{2}$ (d) $\frac{-e^8 - 15}{2}$

10. A unit vector along the vector $4\hat{i} - 3\hat{k}$ is :

- (a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$
(b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$
(c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$
(d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$

11. If θ is the angle between two vectors \vec{a} and \vec{b} , then $\vec{a} \cdot \vec{b} \geq 0$ only when :

- (a) $0 < \theta < \frac{\pi}{2}$ (b) $0 \leq \theta \leq \frac{\pi}{2}$
(c) $0 < \theta < \pi$ (d) $0 \leq \theta \leq \pi$

12. The integrating factor for solving the differential equation $x \frac{dy}{dx} - y = 2x^2$ is :

- (a) e^{-y} (b) e^{-x}
(c) x (d) $\frac{1}{x}$

13. The number of solutions of the differential equation $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(1) = 2$, is :

- (a) zero (b) one
(c) two (d) infinite





14. बिंदु (p, q, r) की y -अक्ष से दूरी है :

- (a) q (b) $|q|$
(c) $|q| + |r|$ (d) $\sqrt{p^2 + r^2}$

15. यदि एक रेखा के दिक्-कोसाइन $\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right)$ हैं, तो :

- (a) $0 < a < 1$ (b) $a > 2$
(c) $a > 0$ (d) $a = \pm\sqrt{3}$

16. दो घटनाओं A तथा B के लिए, यदि $P(A) = 0.4$, $P(B) = 0.8$ तथा $P(B/A) = 0.6$ है, तो $P(A \cup B)$ बराबर है :

- (a) 0.24 (b) 0.3
(c) 0.48 (d) 0.96

17. निम्नलिखित में से कौन-सा बिंदु निम्न दोनों असमिकाओं को संतुष्ट करता है ?

$$2x + y \leq 10 \text{ तथा } x + 2y \geq 8$$

- (a) $(-2, 4)$ (b) $(3, 2)$
(c) $(-5, 6)$ (d) $(4, 2)$

18. असमिका $3x + 5y < 7$ का हल समुच्चय है :

- (a) रेखा $3x + 5y = 7$ पर स्थित बिंदुओं को छोड़कर शेष पूरा xy -तल
(b) रेखा $3x + 5y = 7$ पर स्थित बिंदुओं के साथ पूरा xy -तल
(c) रेखा $3x + 5y = 7$ पर स्थित बिंदुओं को छोड़कर वह खुला आधा तल जिसमें मूल-बिंदु भी है ।
(d) वह खुला आधा तल जिसमें मूल-बिंदु नहीं है ।





14. Distance of the point (p, q, r) from y -axis is :
- (a) q (b) $|q|$
(c) $|q| + |r|$ (d) $\sqrt{p^2 + r^2}$
15. If the direction cosines of a line are $\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right)$, then :
- (a) $0 < a < 1$ (b) $a > 2$
(c) $a > 0$ (d) $a = \pm\sqrt{3}$
16. For two events A and B, if $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$, then $P(A \cup B)$ is :
- (a) 0.24 (b) 0.3
(c) 0.48 (d) 0.96
17. Which of the following points satisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$?
- (a) $(-2, 4)$ (b) $(3, 2)$
(c) $(-5, 6)$ (d) $(4, 2)$
18. The solution set of the inequation $3x + 5y < 7$ is :
- (a) whole xy -plane except the points lying on the line $3x + 5y = 7$.
(b) whole xy -plane along with the points lying on the line $3x + 5y = 7$.
(c) open half plane containing the origin except the points of line $3x + 5y = 7$.
(d) open half plane not containing the origin.





प्रश्न संख्या 19 और 20 अभिकथन एवं तर्क आधारित प्रश्न हैं और प्रत्येक प्रश्न का 1 अंक है। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (a), (b), (c) और (d) में से चुनकर दीजिए।

- (a) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।
- (b) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या नहीं करता है।
- (c) अभिकथन (A) सही है तथा तर्क (R) ग़लत है।
- (d) अभिकथन (A) ग़लत है तथा तर्क (R) सही है।

19. अभिकथन (A) : सभी त्रिकोणमितीय फलनों के अपने प्रान्त में व्युत्क्रम होते हैं।

तर्क (R) : $\tan^{-1} x$ के किसी $x \in \mathbb{R}$ के लिए व्युत्क्रम का अस्तित्व है।

20. अभिकथन (A) : रेखाएँ $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ तथा $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ परस्पर लंबवत् हैं, जब $\vec{b}_1 \cdot \vec{b}_2 = 0$ है।

तर्क (R) : रेखाओं $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ तथा $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ के बीच का कोण θ ,
$$\cos \theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|}$$
 द्वारा प्रदत्त है।

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. वह अंतराल ज्ञात कीजिए जिसमें फलन $f(x) = 2x^3 - 3x$ निरंतर वर्धमान है।





Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
- (c) Assertion (A) is true and Reason (R) is false.
- (d) Assertion (A) is false and Reason (R) is true.

19. Assertion (A) : All trigonometric functions have their inverses over their respective domains.

Reason (R) : The inverse of $\tan^{-1} x$ exists for some $x \in \mathbb{R}$.

20. Assertion (A) : The lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular, when $\vec{b}_1 \cdot \vec{b}_2 = 0$.

Reason (R) : The angle θ between the lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is given by $\cos \theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|}$

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. Find the interval in which the function $f(x) = 2x^3 - 3x$ is strictly increasing.





22. (क) बिन्दु (2, 1, 3) से होकर जाने वाली तथा रेखाओं $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$;
 $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$ दोनों के लंबवत् एक रेखा का सदिश समीकरण ज्ञात कीजिए ।

अथवा

- (ख) एक रेखा के समीकरण $5x - 3 = 15y + 7 = 3 - 10z$ हैं । इस रेखा के दिक्-कोसाइन लिखिए तथा इस पर स्थित एक बिंदु के निर्देशांक ज्ञात कीजिए ।
23. (क) $y = \sin^{-1}(x^2 - 4)$ का प्रान्त ज्ञात कीजिए ।

अथवा

- (ख) मान ज्ञात कीजिए :

$$\cos^{-1} \left[\cos \left(-\frac{7\pi}{3} \right) \right]$$

24. यदि $(x^2 + y^2)^2 = xy$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए ।

25. यदि $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ तथा $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ है, तो सदिश $\vec{a} \times \vec{b}$ की दिशा में एक मात्रक सदिश ज्ञात कीजिए ।

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं ।

26. ज्ञात कीजिए :

$$\int \frac{x^2}{x^2 + 6x + 12} dx$$

27. दो अनभिन्नत पासे एक साथ उछाले गए । यदि X, प्राप्त हुए छः की संख्या को दर्शाता है, तो X का माध्य ज्ञात कीजिए ।

28. (क) अवकल समीकरण $\frac{dy}{dx} = \frac{x+y}{x}$, $y(1) = 0$ का विशिष्ट हल ज्ञात कीजिए ।

अथवा

- (ख) अवकल समीकरण $e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$ का व्यापक हल ज्ञात कीजिए ।





22. (a) Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines

$$\frac{x - 1}{1} = \frac{y - 2}{2} = \frac{z - 3}{3}; \quad \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}.$$

OR

- (b) The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line and find the coordinates of a point through which it passes.
23. (a) Find the domain of $y = \sin^{-1}(x^2 - 4)$.

OR

- (b) Evaluate :

$$\cos^{-1} \left[\cos \left(-\frac{7\pi}{3} \right) \right]$$

24. If $(x^2 + y^2)^2 = xy$, then find $\frac{dy}{dx}$.

25. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector along the vector $\vec{a} \times \vec{b}$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{x^2}{x^2 + 6x + 12} dx$$

27. Two fair dice are thrown simultaneously. If X denotes the number of sixes, find the mean of X.

28. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{x+y}{x}, \quad y(1) = 0.$$

OR

- (b) Find the general solution of the differential equation $e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$.





29. (क) मान ज्ञात कीजिए :

$$\int_{\pi/4}^{\pi/2} e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_{-2}^2 \frac{x^2}{1 + 5^x} dx$$

30. (क) ज्ञात कीजिए :

$$\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_0^{\pi/2} \sqrt{\sin x} \cos^5 x dx$$

31. निम्नलिखित रैखिक प्रोग्रामन समस्या को आलेख द्वारा हल कीजिए :
व्यवरोधों

$$-2x + y \leq 4,$$

$$x + y \geq 3,$$

$$x - 2y \leq 2,$$

$$x \geq 0, y \geq 0$$

के अंतर्गत $z = -3x - 5y$ का अधिकतम मान ज्ञात कीजिए ।





29. (a) Evaluate :

$$\int_{\pi/4}^{\pi/2} e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

OR

(b) Evaluate :

$$\int_{-2}^2 \frac{x^2}{1 + 5^x} dx$$

30. (a) Find :

$$\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/2} \sqrt{\sin x} \cos^5 x dx$$

31. Solve the following linear programming problem graphically :

Maximise $z = -3x - 5y$

subject to the constraints

$$-2x + y \leq 4,$$

$$x + y \geq 3,$$

$$x - 2y \leq 2,$$

$$x \geq 0, y \geq 0.$$





खण्ड घ

इस खण्ड में दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. समाकलन के प्रयोग से, वृत्त $x^2 + y^2 = 16$, रेखा $y = x$ तथा y -अक्ष द्वारा प्रथम चतुर्थांश में घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

33. (क) दर्शाइए कि निम्नलिखित रेखाएँ परस्पर प्रतिच्छेदी नहीं हैं :

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}; \quad \frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$$

अथवा

(ख) रेखाओं $2x = 3y = -z$ तथा $6x = -y = -4z$ के बीच का कोण ज्ञात कीजिए।

34. (क) यदि N , सभी प्राकृत संख्याओं के समुच्चय को निर्धारित करता है तथा $N \times N$ में एक संबंध R , इस प्रकार परिभाषित है कि $(a, b) R (c, d)$, यदि $ad(b+c) = bc(a+d)$. दर्शाइए कि R एक तुल्यता संबंध है।

अथवा

(ख) माना $f: \mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbb{R}$, $f(x) = \frac{4x}{3x+4}$ द्वारा परिभाषित एक फलन है। दर्शाइए कि f एक एकैकी फलन है। यह भी जाँच कीजिए कि f एक आच्छादक फलन है या नहीं।

35. आव्यूह $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ का व्युत्क्रम ज्ञात कीजिए। व्युत्क्रम A^{-1} के प्रयोग से,

रैखिक समीकरण निकाय $x - y + 2z = 1$; $2y - 3z = 1$; $3x - 2y + 4z = 3$ को हल कीजिए।





SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. Using integration, find the area of the region bounded by the circle $x^2 + y^2 = 16$, line $y = x$ and y -axis, but lying in the 1st quadrant.

33. (a) Show that the following lines do not intersect each other :

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}; \quad \frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$$

OR

(b) Find the angle between the lines

$$2x = 3y = -z \text{ and } 6x = -y = -4z.$$

34. (a) If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by $(a, b) R (c, d)$, if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

OR

(b) Let $f: \mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show

that f is a one-one function. Also, check whether f is an onto function or not.

35. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse,

A^{-1} , solve the system of linear equations

$$x - y + 2z = 1; \quad 2y - 3z = 1; \quad 3x - 2y + 4z = 3.$$





खण्ड ड

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन - 1

36. एक भवन बनाने वाला ठेकेदार, एक प्लॉट पर 4 फ्लैट तथा पार्किंग क्षेत्र का काम लेता है। हड़ताल के कारण बहुत से श्रमिकों के काम पर न आ पाने की प्रायिकता 0.65 है। बहुत से श्रमिकों के न होने पर भी कार्य के समय पर पूरा हो जाने की प्रायिकता 0.35 है। सभी श्रमिकों के काम पर आने पर कार्य समय पर पूरा होने की प्रायिकता 0.80 है।

माना : E_1 : निरूपित करता है उस घटना को जब बहुत से श्रमिक काम पर नहीं आए;

E_2 : निरूपित करता है वह घटना जब सभी श्रमिक काम पर आए; और

E : निरूपित करता है कि कार्य समय पर पूरा हो जाता है।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

- (i) सभी श्रमिकों के काम पर आने की प्रायिकता क्या है ? 1
- (ii) कार्य समय पर पूरा हो जाने की प्रायिकता क्या है ? 1
- (iii) (क) दिया गया है कि कार्य समय पर पूरा हो गया, तो बहुत से श्रमिकों के काम पर न आने की प्रायिकता क्या है ? 2

अथवा

- (iii) (ख) दिया गया है कि कार्य समय पर पूरा हो गया, तो सभी श्रमिकों के काम पर उपस्थित होने की प्रायिकता क्या है ? 2





SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

- 36.** A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let : E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

- (i) What is the probability that all the workers are present for the job ? 1
- (ii) What is the probability that construction will be completed on time ? 1
- (iii) (a) What is the probability that many workers are not present given that the construction work is completed on time ? 2

OR

- (iii) (b) What is the probability that all workers were present given that the construction job was completed on time ? 2





प्रकरण अध्ययन - 2

37. माना $f(x)$ एक वास्तविक मान वाला फलन है। तो इसका

- बाएँ पक्ष का अवकलज (L.H.D.) : $Lf'(a) = \lim_{h \rightarrow 0} \frac{f(a-h) - f(a)}{-h}$
- दाएँ पक्ष का अवकलज (R.H.D.) : $Rf'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

साथ ही, एक फलन $f(x)$, $x = a$ पर अवकलनीय कहलाता है यदि $x = a$ पर इसके L.H.D.

और R.H.D. का अस्तित्व है तथा दोनों समान हैं।

$$\text{फलन } f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$$

के लिए निम्नलिखित प्रश्नों के उत्तर दीजिए :

- (i) $f(x)$ का $x = 1$ पर दाएँ पक्ष का अवकलज (R.H.D.) क्या है ? 1
- (ii) $f(x)$ का $x = 1$ पर बाएँ पक्ष का अवकलज (L.H.D.) क्या है ? 1
- (iii) (क) जाँच कीजिए कि क्या $x = 1$ पर फलन $f(x)$ अवकलनीय है। 2

अथवा

- (iii) (ख) $f'(2)$ तथा $f'(-1)$ ज्ञात कीजिए। 2





Case Study - 2

37. Let $f(x)$ be a real valued function. Then its

- Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim_{h \rightarrow 0} \frac{f(a - h) - f(a)}{-h}$
- Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$

Also, a function $f(x)$ is said to be differentiable at $x = a$ if its L.H.D. and R.H.D. at $x = a$ exist and both are equal.

$$\text{For the function } f(x) = \begin{cases} |x - 3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$$

answer the following questions :

- (i) What is R.H.D. of $f(x)$ at $x = 1$? 1
- (ii) What is L.H.D. of $f(x)$ at $x = 1$? 1
- (iii) (a) Check if the function $f(x)$ is differentiable at $x = 1$. 2

OR

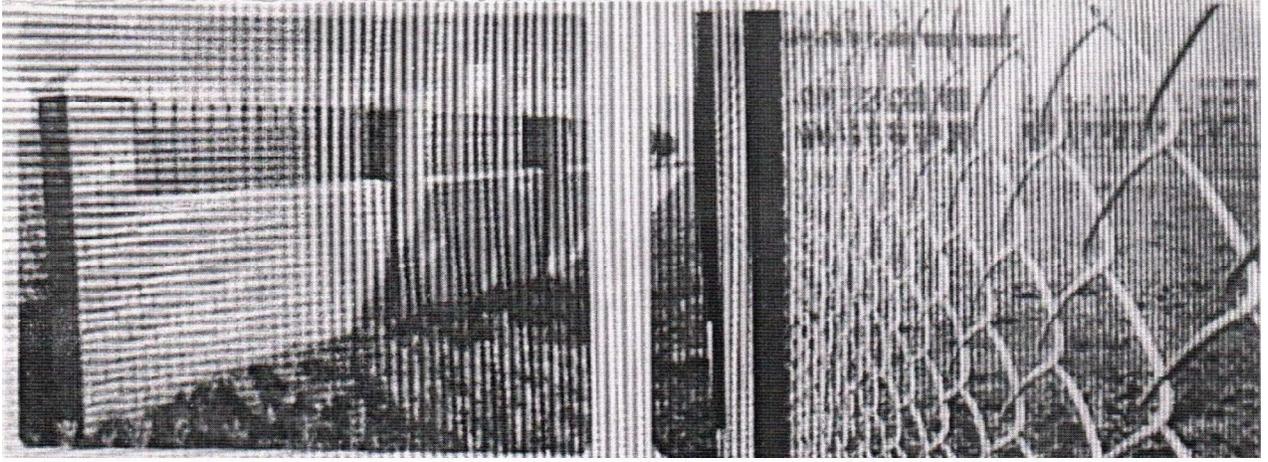
- (iii) (b) Find $f'(2)$ and $f'(-1)$. 2





प्रकरण अध्ययन – 3

38. सूरज के पिता एक ईंटों की दीवार को एक साइड लेकर, एक आयताकार बाग बनाना चाहते हैं जिसकी बाकी तीन साइडों पर तार की बाड़ लगाना चाहते हैं (जैसा चित्र में दिखाया है) । उसके पास बाड़ के लिए 200 मीटर की तार है ।



उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

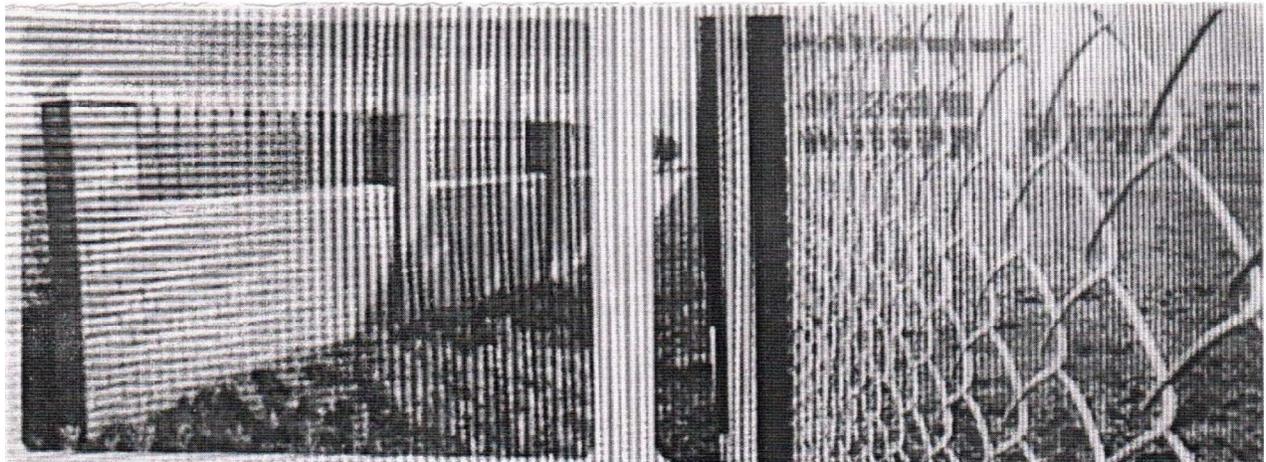
- (i) माना बाग में ईंट की दीवार के लंबवत् साइड की लंबाई 'x' मीटर है तथा ईंट की दीवार के समांतर साइड की लंबाई 'y' मीटर है । कुल लगने वाली तार की लंबाई का सूत्र (संबंध) ज्ञात कीजिए तथा बाग का क्षेत्रफल $A(x)$ भी लिखिए । 2
- (ii) $A(x)$ का अधिकतम मान ज्ञात कीजिए । 2





Case Study – 3

38. Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.



Based on the above information, answer the following questions :

- (i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write $A(x)$, the area of the garden. 2
- (ii) Determine the maximum value of $A(x)$. 2



Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2023
MATHEMATICS PAPER CODE 65/4/2

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc. may invite action under various rules of the Board and IPC.”
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (\surd) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (\checkmark) while evaluating which gives the impression that answer is correct, and no marks are awarded. This is most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note “Extra Question”.</u>

10	<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question”.</u>
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) must be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - <ul style="list-style-type: none"> ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the “ Guidelines for spot Evaluation ” before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME

Senior Secondary School Examination, 2022-23

MATHEMATICS (Subject Code-041)

(Paper Code-65/4/2)

Section A

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each .	
1.	If $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$, then the value of $(2x + y - z)$ is : (a) 1 (b) 2 (c) 3 (d) 5	
Sol.	(d)5	1
2.	If a matrix $A = [1 \ 2 \ 3]$, then the matrix AA' (where A' is the transpose of A) is : (a) 14 (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (d) $[14]$	
Sol.	(d)[14]	1
3.	If $x \begin{bmatrix} 1 \\ 2 \end{bmatrix} + y \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$, then : (a) $x = 1, y = 2$ (b) $x = 2, y = 1$ (c) $x = 1, y = -1$ (d) $x = 3, y = 2$	

12.	The integrating factor for solving the differential equation $x \frac{dy}{dx} - y = 2x^2$ is : (a) e^{-y} (b) e^{-x} (c) x (d) $\frac{1}{x}$	
Sol.	(d) $\frac{1}{x}$	1
13.	The number of solutions of the differential equation $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(1) = 2$, is : (a) zero (b) one (c) two (d) infinite	
Sol.	(b) one	1
14.	Distance of the point (p, q, r) from y-axis is : (a) q (b) q (c) q + r (d) $\sqrt{p^2 + r^2}$	
Sol.	(d) $\sqrt{p^2 + r^2}$	1
15.	If the direction cosines of a line are $\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right)$, then : (a) $0 < a < 1$ (b) $a > 2$ (c) $a > 0$ (d) $a = \pm \sqrt{3}$	
Sol.	(a) $a = 2$	1
16.	For two events A and B, if $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$, then $P(A \cup B)$ is : (a) 0.24 (b) 0.3 (c) 0.48 (d) 0.96	
Sol.	(d) 0.96	1

17.	Which of the following points satisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$? (a) $(-2, 4)$ (b) $(3, 2)$ (c) $(-5, 6)$ (d) $(4, 2)$	
Sol.	(d)(4, 2)	1
18.	The solution set of the inequation $3x + 5y < 7$ is : (a) whole xy-plane except the points lying on the line $3x + 5y = 7$. (b) whole xy-plane along with the points lying on the line $3x + 5y = 7$. (c) open half plane containing the origin except the points of line $3x + 5y = 7$. (d) open half plane not containing the origin.	
Sol.	(c) open half plane containing the origin except the points of line $3x + 5y = 7$	1
	<i>Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.</i> (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (c) Assertion (A) is true and Reason (R) is false. (d) Assertion (A) is false and Reason (R) is true.	
19.	<i>Assertion (A) :</i> All trigonometric functions have their inverses over their respective domains. <i>Reason (R) :</i> The inverse of $\tan^{-1} x$ exists for some $x \in \mathbb{R}$.	
Sol.	(d) Assertion (A) is false and Reason (R) is true.	1

20.	<p><i>Assertion (A)</i> : The lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular, when $\vec{b}_1 \cdot \vec{b}_2 = 0$.</p> <p><i>Reason (R)</i> : The angle θ between the lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is given by $\cos \theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{ \vec{b}_1 \vec{b}_2 }$</p>	
Sol.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)	1
21.	Find the interval in which the function $f(x) = 2x^3 - 3x$ is strictly increasing.	
Sol.	<p>Here, $f(x) = 2x^3 - 3x$</p> $f'(x) = 6x^2 - 3 = 3(2x^2 - 1)$ <p>Now, $f'(x) = 0 \Rightarrow x = \pm \frac{1}{\sqrt{2}}$</p> <p>$f(x)$ is strictly increasing in $(-\infty, -\frac{1}{\sqrt{2}}) \cup (\frac{1}{\sqrt{2}}, \infty)$</p>	$\frac{1}{2}$ 1 $\frac{1}{2}$
22.	<p>(a) Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines</p> $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}; \quad \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$ <p style="text-align: center;">OR</p> <p>(b) The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line and find the coordinates of a point through which it passes.</p>	
Sol.	<p>(a) Vector equation of the line passing through (2, 1, 3) is</p> $\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$ <p>Line \vec{r} is perpendicular to the given lines then</p>	$\frac{1}{2}$

$$a + 2b + 3c = 0; \quad -3a + 2b + 5c = 0$$

$$\Rightarrow \frac{a}{4} = \frac{b}{-14} = \frac{c}{8} = k' \text{ (say)}$$

$$\Rightarrow a = 2k, \quad b = -7k \text{ and } c = 4k.$$

Thus, the required vector equation is

$$\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$$

OR

(b) The equation of the given line is

$$\frac{x - 3/5}{1/5} = \frac{y + 7/15}{1/15} = \frac{z - 3/10}{-1/10}$$

Its direction ratios are

$$\left(\frac{1}{5}, \frac{1}{15}, -\frac{1}{10}\right) \text{ or } (6, 2, -3)$$

$$\left\langle \frac{1}{5}, \frac{1}{15}, -\frac{1}{10} \right\rangle \text{ or } \langle 6, 2, -3 \rangle$$

$$\text{Direction cosines are } \left\langle \pm \frac{6}{7}, \pm \frac{2}{7}, \mp \frac{3}{7} \right\rangle$$

$$\text{The point through which it passes is } \left(\frac{3}{5}, \frac{-7}{15}, \frac{3}{10}\right)$$

1

$\frac{1}{2}$

1

$\frac{1}{2}$

$\frac{1}{2}$

23. (a) Find the domain of $y = \sin^{-1}(x^2 - 4)$.

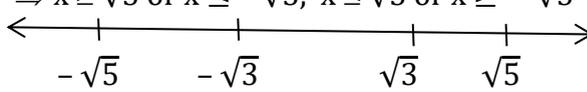
OR

(b) Evaluate :

$$\cos^{-1} \left[\cos \left(-\frac{7\pi}{3} \right) \right]$$

Sol.

(a) Domain of $\sin^{-1} x$ is $-1 \leq x \leq 1$

	$\therefore -1 \leq x^2 - 4 \leq 1$ $\text{or } x^2 \geq 3, x^2 \leq 5$ $\Rightarrow x \geq \sqrt{3} \text{ or } x \leq -\sqrt{3}, x \leq \sqrt{5} \text{ or } x \geq -\sqrt{5}$  $\therefore \text{Domain is } [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$ <p style="text-align: center;">OR</p> <p>(b) $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right] = \cos^{-1}\left[\cos\left(\frac{7\pi}{3}\right)\right]$</p> $= \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{3}\right)\right]$ $= \cos^{-1}\left[\cos\frac{\pi}{3}\right] = \frac{\pi}{3}$	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$
24.	If $(x^2 + y^2)^2 = xy$, then find $\frac{dy}{dx}$.	
Sol.	$(x^2 + y^2)^2 = xy$ gives $2(x^2 + y^2) \left[2x + 2y \frac{dy}{dx}\right] = x \frac{dy}{dx} + y$ $\Rightarrow [4y(x^2 + y^2) - x] \frac{dy}{dx} = y - 4x(x^2 + y^2)$ $\Rightarrow \frac{dy}{dx} = \frac{y - 4x(x^2 + y^2)}{4y(x^2 + y^2) - x}$	1 $\frac{1}{2}$ $\frac{1}{2}$
25.	If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector along the vector $\vec{a} \times \vec{b}$.	
Sol.	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = \hat{i} - 2\hat{j} - 6\hat{k}$	1

$$I = x - 3 \log |x^2 + 6x + 12| + 2\sqrt{3} \tan^{-1} \left(\frac{x+3}{\sqrt{3}} \right) + C$$

$$\frac{1}{2}$$

27. Two fair dice are thrown simultaneously. If X denotes the number of sixes, find the mean of X.

Sol. Here X can take values 0, 1 or 2

$$P(X = 0) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$$

$$P(X = 1) = 2 \left(\frac{5}{6} \times \frac{1}{6} \right) = \frac{10}{36}$$

$$P(X = 2) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$\text{Mean of X} = \left(0 \times \frac{25}{36} \right) + \left(1 \times \frac{10}{36} \right) + \left(2 \times \frac{1}{36} \right)$$

$$= \frac{12}{36} \text{ or } \frac{1}{3}$$

$$\frac{1}{2}$$

$$1\frac{1}{2}$$

$$1$$

28. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{x+y}{x}, \quad y(1) = 0.$$

OR

- (b) Find the general solution of the differential equation
 $e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0.$

Sol. (a) $\frac{dy}{dx} = \frac{x+y}{x} \Rightarrow \frac{dy}{dx} = 1 + \frac{y}{x}$

Let $\frac{y}{x} = v$. Then $x \frac{dv}{dx} + v = \frac{dy}{dx}$

So, D.E. becomes

$$x \frac{dv}{dx} + v = 1 + v$$

$$\Rightarrow dv = \frac{dx}{x}$$

$$1$$

$$\Rightarrow v = \log|x| + c$$

$$\Rightarrow y = x \log|x| + cx$$

$$\Rightarrow x = 1, y = 0 \Rightarrow c = 0, y = x \log|x|$$

[can also be solved, taking as first order linear diff. eqn]

OR

(b) The given D.E. is

$$\frac{\sec^2 y}{\tan y} dy = -\frac{e^x}{1 - e^x} dx$$

Integrating

$$\Rightarrow \log|\tan y| = \log|1 - e^x| + \log C$$

$$\Rightarrow \tan y = C(1 - e^x)$$

1

1

1

1

1

29.

(a) Evaluate :

$$\int_{\pi/4}^{\pi/2} e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

OR

(b) Evaluate :

$$\int_{-2}^2 \frac{x^2}{1 + 5^x} dx$$

Sol.

$$(a) I = \int_{\pi/4}^{\pi/2} e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

Put $2x = t$ so that $2 dx = dt$

$$\text{When } x = \frac{\pi}{2}, t = \pi, x = \frac{\pi}{4}, t = \frac{\pi}{2}$$

Thus

$$I = \int_{\pi/2}^{\pi} e^t \left(\frac{1 - \sin t}{1 - \cos t} \right) \frac{dt}{2}$$

$\frac{1}{2}$

$$= \int_{\pi/2}^{\pi} e^t \left(\frac{1 - 2 \sin t/2 \cos t/2}{2 \sin^2 t/2} \right) \frac{dt}{2}$$

$$= \frac{1}{2} \int_{\pi/2}^{\pi} e^t \left(\frac{1}{2} \operatorname{cosec}^2 \frac{t}{2} - \cot \frac{t}{2} \right) dt$$

$$= -\frac{1}{2} \left| e^t \cot \frac{t}{2} \right|_{\pi/2}^{\pi}$$

$$= -\frac{1}{2} \left| e^{\pi} \cot \frac{\pi}{2} - e^{\pi/2} \cot \frac{\pi}{4} \right|$$

$$= \frac{1}{2} e^{\pi/2}$$

OR

(b) $I = \int_{-2}^2 \frac{x^2}{1+5^x} dx$ ----- (1)

Replacing x by -2+2-x

$$= \int_{-2}^2 \frac{(-2+2-x)^2}{1+5^{(-2+2-x)}} dx$$

$$I = \int_{-2}^2 \frac{x^2 5^x}{1+5^x} dx$$
 ----- (2)

Adding (1) + (2)

$$2I = \int_{-2}^2 \frac{x^2(1+5^x)}{1+5^x} dx = \int_{-2}^2 x^2 dx = \left| \frac{x^3}{3} \right|_{-2}^2 = \frac{8}{3} + \frac{8}{3} = \frac{16}{3}$$

$$\Rightarrow I = \frac{16}{6} \text{ or } \frac{8}{3}$$

1

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

1

$\frac{1}{2}$

1

$\frac{1}{2}$

30.	<p>(a) Find :</p> $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx$ <p style="text-align: center;">OR</p> <p>(b) Evaluate :</p> $\int_0^{\pi/2} \sqrt{\sin x} \cos^5 x dx$	
Sol.	<p>(a) Let $e^x = t$, so that $e^x dx = dt$. Then,</p> $I = \int \frac{dt}{\sqrt{5-4t-t^2}} = \int \frac{dt}{\sqrt{3^2-(t+2)^2}}$ $= \sin^{-1} \left(\frac{t+2}{3} \right) + C$ $= \sin^{-1} \left(\frac{e^x+2}{3} \right) + C$ <p style="text-align: center;">OR</p> <p>(b) $I = \int_0^{\pi/2} \sqrt{\sin x} \cdot (1 - \sin^2 x)^2 \cos x dx$</p> <p>Put $\sin x = t^2$</p> $= \int_0^1 \sqrt{t} (1 - t^2)^2 dt = \int_0^1 (\sqrt{t} + t^{9/2} - 2t^{5/2}) dt$ $= \left[\frac{2t^{3/2}}{3} + \frac{2t^{11/2}}{11} - \frac{4t^{7/2}}{7} \right]_0^1 = \frac{2}{3} + \frac{2}{11} - \frac{4}{7} = \frac{64}{231}$	<p style="text-align: center;">$\frac{1}{2}$</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">$\frac{1}{2}$</p> <p style="text-align: center;">$\frac{1}{2}$</p> <p style="text-align: center;">1</p> <p style="text-align: center;">$1 + \frac{1}{2}$</p>

31.

Solve the following linear programming problem graphically :

Maximise $z = -3x - 5y$

subject to the constraints

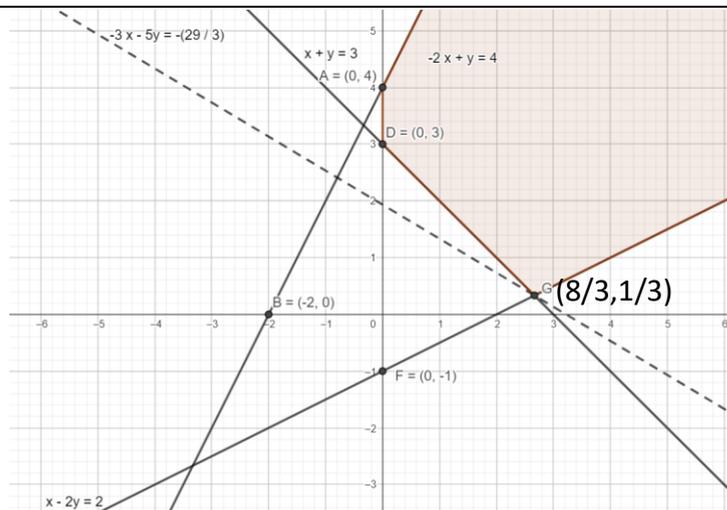
$$-2x + y \leq 4,$$

$$x + y \geq 3,$$

$$x - 2y \leq 2,$$

$$x \geq 0, y \geq 0.$$

Sol.



Corner Points	Value of Z
A (0,4)	-20
D (0,3)	-15
G (8/3,1/3)	$-29/3 \rightarrow \text{Max}$

Since feasible region is unbounded and $-\frac{29}{3}$ is the maximum value of z at corner $\left(\frac{8}{3}, \frac{1}{3}\right)$. So, we consider the open half plane $-3x - 5y > -\frac{29}{3}$, which

2 for
correct
graph

has no point common to the feasible region. Hence $\text{Max } z = -\frac{29}{3}$, which occurs at $(\frac{8}{3}, \frac{1}{3})$.

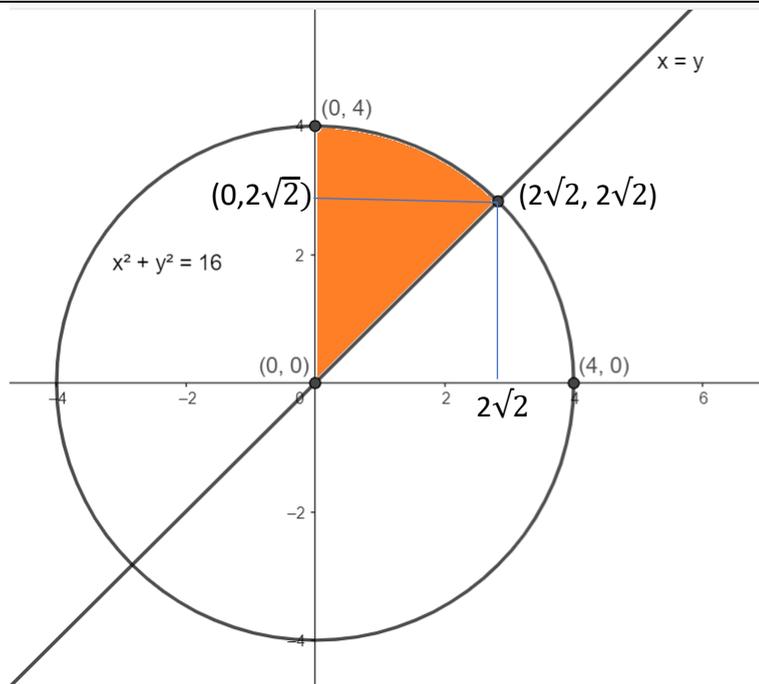
1

SECTION D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32. Using integration, find the area of the region bounded by the circle $x^2 + y^2 = 16$, line $y = x$ and y-axis, but lying in the 1st quadrant.

Sol.



$$\text{Required area} = \int_0^{2\sqrt{2}} y \, dy + \int_{2\sqrt{2}}^4 \sqrt{16 - y^2} \, dy$$

$$= \left| \frac{y^2}{2} \right|_0^{2\sqrt{2}} + \left| \frac{y}{2} \sqrt{16 - y^2} + \frac{16}{2} \sin^{-1} \left(\frac{y}{4} \right) \right|_{2\sqrt{2}}^4$$

$$= 4 + \left\{ 8 \sin^{-1}(1) - \sqrt{2} \cdot \sqrt{8} - 8 \sin^{-1} \left(\frac{1}{\sqrt{2}} \right) \right\}$$

$$= 4 + 8 \left(\frac{\pi}{2} \right) - 4 - 8 \left(\frac{\pi}{4} \right)$$

1

1

$\frac{1}{2}$

$\frac{1}{2}$

$$= 8 \left(\frac{\pi}{4} \right) \text{ or } 2\pi$$

1

33. (a) Show that the following lines do not intersect each other :

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}; \quad \frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$$

OR

(b) Find the angle between the lines

$$2x = 3y = -z \text{ and } 6x = -y = -4z.$$

Sol.

$$\begin{aligned} \text{(a) Shortest Distance} &= \frac{\begin{vmatrix} -3 & 2 & -2 \\ 3 & 2 & 5 \\ 4 & 3 & -2 \end{vmatrix}}{\sqrt{(2 \times -2 - 5 \times 3)^2 + (3 \times -2 - 4 \times 5)^2 + (3 \times 3 - 2 \times 4)^2}} \\ &= \frac{-3(-19) - 2(-26) - 2(1)}{\sqrt{361 + 676 + 1}} \\ &= \frac{57 + 52 - 2}{\sqrt{1038}} \\ &= \frac{107}{\sqrt{1038}} \neq 0 \end{aligned}$$

So, the line will not intersect each other.

OR

(b) The given lines are

$$\frac{x-0}{1/2} = \frac{y-0}{1/3} = \frac{z-0}{-1} \text{ and } \frac{x-0}{1/6} = \frac{y-0}{-1} = \frac{z-0}{-1/4}$$

$$\text{Or } \frac{x}{3} = \frac{y}{2} = \frac{z}{-6} \text{ and } \frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$$

Let θ be the angle between the two lines, then

$$\begin{aligned} \cos \theta &= \left| \frac{(3 \times 2) + (2 \times -12) + (-6)(-3)}{\sqrt{9 + 4 + 36} \sqrt{4 + 144 + 9}} \right| \\ &= \left| \frac{6 - 24 + 18}{7 \times \sqrt{157}} \right| \end{aligned}$$

2

1

1

1

1+1

2

$$= 0$$

$$\Rightarrow \theta = 90^\circ$$

1

34. (a) If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by $(a, b) R (c, d)$, if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

OR

(b) Let $f : \mathbb{R} - \left\{ -\frac{4}{3} \right\} \rightarrow \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x + 4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

Sol. (a) Reflexive : Here, $(a, b) R (a, b) \forall (a, b) \in N \times N$ since $ab(b + a) = ba(a + b)$ is always true.

Symmetric : Let $(a, b) R (c, d) \forall (a, b), (c, d) \in N \times N$. Then,

$$ad(b + c) = bc(a + d)$$

$$\Rightarrow bc(a + d) = ad(b + c)$$

$$\Rightarrow (c, d) R (a, b)$$

Transitive : Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$.

$\forall (a, b), (c, d), (e, f) \in N \times N$ Then

$$ad(b + c) = bc(a + d) \text{ and } cf(d + e) = de(c + f)$$

$$\Rightarrow \frac{b + c}{bc} = \frac{a + d}{ad} \text{ and } \frac{d + e}{de} = \frac{c + f}{cf}$$

$$\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a} \Rightarrow \frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$$

Adding, we get

$$\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$$

$1\frac{1}{2}$
(for symmetric)

2
(for transitive)

$$\Rightarrow \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f}$$

$$\Rightarrow \frac{e + b}{be} = \frac{f + a}{af}$$

$$\Rightarrow af(b + e) = be(a + f)$$

$$\Rightarrow (a, b) R (e, f)$$

Hence, R is equivalence relation.

OR

(b) For one-one

$$\text{Let } f(x_1) = f(x_2) \text{ for some } x_1, x_2 \in R - \left\{ -\frac{4}{3} \right\}$$

$$\Rightarrow \frac{4x_1}{3x_1 + 4} = \frac{4x_2}{3x_2 + 4}$$

$$\Rightarrow 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$$

$$\Rightarrow 16x_1 = 16x_2$$

$$\Rightarrow x_1 = x_2$$

$$\Rightarrow f \text{ is one-one}$$

For onto let $y \in R$, and for some x .

$$\text{Let } y = \frac{4x}{3x + 4}$$

$$\Rightarrow 3xy + 4y = 4x$$

$$\Rightarrow x(3y - 4) = -4y$$

$$\Rightarrow x = -\frac{4y}{3y - 4} \text{ or } x = \frac{4y}{4 - 3y}$$

x is real if $y \neq \frac{4}{3}$. So $R_f = R - \left\{ \frac{4}{3} \right\} \neq \text{Codomain}(f)$

$\frac{1}{2}$

1

1

1

1

So, f is not onto.

1

35.

Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse,

A^{-1} , solve the system of linear equations

$$x - y + 2z = 1; 2y - 3z = 1; 3x - 2y + 4z = 3.$$

Sol.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

$$|A| = 1(8 - 6) + 1(0 + 9) + 2(0 - 6) = -1 \neq 0$$

$\therefore A$ is invertible.

$$\text{adj } A = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj } A = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

The given system of equation can be written as $AX = B$, when

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow X = A^{-1} B$$

1

$1\frac{1}{2}$

$1\frac{1}{2}$

1

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow x = 1, y = 2, z = 1$$

1

SECTION E

This section comprises of 3 case-study based questions of 4 marks each.

36.

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let : E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

- (i) What is the probability that all the workers are present for the job ? 1
- (ii) What is the probability that construction will be completed on time ? 1
- (iii) (a) What is the probability that many workers are not present given that the construction work is completed on time ? 2

OR

- (iii) (b) What is the probability that all workers were present given that the construction job was completed on time ? 2

Sol.

$$(i) P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$$

$$(ii) P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$$

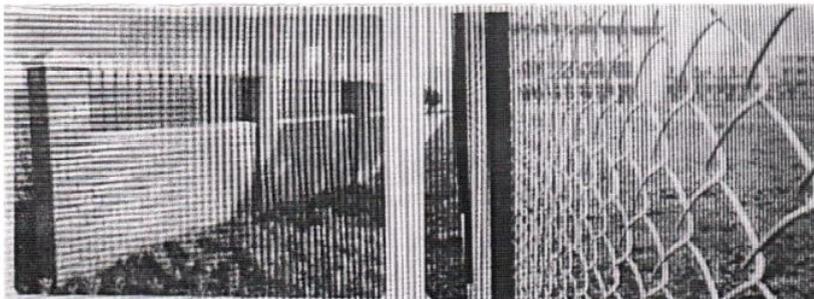
$$= 0.65 \times 0.35 + 0.35 \times 0.8$$

$$= 0.35 \times 1.45$$

1

	$= 0.51$ <p>(iii) (a) $P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)} = \frac{0.65 \times 0.35}{0.51} = 0.45$</p> <p style="text-align: center;">OR</p> <p>(iii) (b) $P\left(\frac{E_2}{E}\right) = \frac{P(E_2) \cdot P(E/E_2)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)} = \frac{0.35 \times 0.8}{0.51} = 0.55$</p>	1 2 2
37.	<p>Let $f(x)$ be a real valued function. Then its</p> <ul style="list-style-type: none"> • Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim_{h \rightarrow 0} \frac{f(a-h) - f(a)}{-h}$ • Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ <p>Also, a function $f(x)$ is said to be differentiable at $x = a$ if its L.H.D. and R.H.D. at $x = a$ exist and both are equal.</p> <p>For the function $f(x) = \begin{cases} x-3 , & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$</p> <p>answer the following questions :</p> <p>(i) What is R.H.D. of $f(x)$ at $x = 1$? 1</p> <p>(ii) What is L.H.D. of $f(x)$ at $x = 1$? 1</p> <p>(iii) (a) Check if the function $f(x)$ is differentiable at $x = 1$. 2</p> <p style="text-align: center;">OR</p> <p>(iii) (b) Find $f'(2)$ and $f'(-1)$. 2</p>	
Sol.	<p>(i) R.H.D. of $f(x)$ at $x = 1$ is $= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$</p> $= \lim_{h \rightarrow 0} \frac{ 1+h-3 - -2 }{h} = \lim_{h \rightarrow 0} \frac{2-h-2}{h} = -1$ <p>(ii) L.H.D. of $f(x)$ at $x = 1 = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h}$</p>	1

	$= \lim_{h \rightarrow 0} \frac{-1}{h} \left[\frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4} - 2 \right]$ $= \lim_{h \rightarrow 0} \left[\frac{h^2 - 2h + 1 - 6 + 6h + 13 - 8}{-4h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{h^2 + 4h}{-4h} \right] = -1.$ <p>(iii) (a) Since L.H.D. of $f(x)$ at $x = 1$ = R.H.D. of $f(x)$ at $x = 1$, $f(x)$ is differentiable at $x = 1$.</p> <p style="text-align: center;">OR</p> <p>(iii) (b) $f(x) = \begin{cases} x - 3, & x \geq 3 \\ 3 - x, & 1 \leq x < 3 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$</p> $[f'(x)]_{x=2} = 0 - 1 = -1$ $[f'(x)]_{x=-1} = \frac{2(-1)}{4} - \frac{3}{2} = -2$	<p style="text-align: center;">1</p> <p style="text-align: center;">2</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p>
38.	<p>Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.</p>	

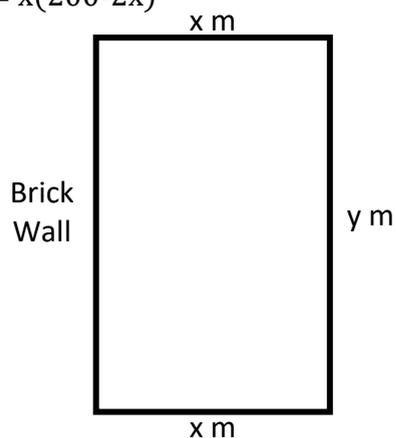


Based on the above information, answer the following questions :

- (i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden. 2
- (ii) Determine the maximum value of A(x). 2

Sol. (i) (a) $2x + y = 200$

(b) $A(x) = xy = x(200 - 2x)$



(ii) From (a) and (b) of (i) we have

$$A(x) = x(200 - 2x)$$

$$= 200x - 2x^2$$

From max./min of A(x)

$$\frac{dA}{dx} = 0 \quad \text{i.e. } 200 - 4x = 0$$

1
1

$$\Rightarrow x = 50.$$

$$\frac{d^2A}{dx^2} = -4$$

$$\left(\frac{d^2A}{dx^2}\right)_{x=50} < 0$$

Hence, A(x) is maximum at x = 50

$$\text{Thus, Max } A(x) = 200(50) - 2(50)^2$$

$$= 10000 - 5000$$

$$= 5000 \text{ sqm.}$$

1

1