

Series PQ1RS/1

Set – 1



प्रश्न-पत्र कोड
Q.P. Code

65/1/1

अनुक्रमांक

Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 23 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please write down the serial number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.



गणित
MATHEMATICS



निर्धारित समय : 3 घण्टे

अधिकतम अंक : 80

Time allowed : 3 hours

Maximum Marks : 80



सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) इस प्रश्न-पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- (ii) यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – क, ख, ग, घ एवं ङ।
- (iii) खण्ड क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
- (iv) खण्ड ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं।
- (v) खण्ड ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं।
- (vi) खण्ड घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
- (vii) खण्ड ङ में प्रश्न संख्या 36 से 38 प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं।
- (viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
- (ix) कैल्कुलेटर का उपयोग वर्जित है।

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

1. एक फलन $f: \mathbb{R}_+ \rightarrow \mathbb{R}$ (जहाँ \mathbb{R}_+ सभी ऋणेतर वास्तविक संख्याओं का समुच्चय है) $f(x) = 4x + 3$ द्वारा परिभाषित है, तो यह फलन :
 - (A) एकैकी है परन्तु आच्छादक नहीं है
 - (B) आच्छादक है परन्तु एकैकी नहीं है
 - (C) एकैकी तथा आच्छादक दोनों हैं
 - (D) न तो एकैकी और न ही आच्छादक है
2. यदि एक आव्यूह के 36 अवयव हैं, तो इसकी संभव कोटियों की संख्या है :
 - (A) 13
 - (B) 3
 - (C) 5
 - (D) 9



General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (ix) Use of calculators is **not** allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. A function $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ (where \mathbb{R}_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is :
 - (A) one-one but not onto
 - (B) onto but not one-one
 - (C) both one-one and onto
 - (D) neither one-one nor onto

2. If a matrix has 36 elements, the number of possible orders it can have, is :
 - (A) 13
 - (B) 3
 - (C) 5
 - (D) 9



3. फलन $f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$ के लिए, निम्न कथनों में से कौन-सा सही है ?
- (A) $f(x)$ संतत और अवकलनीय है, सभी $x \in \mathbb{R}$ के लिए
(B) $f(x)$ संतत है, सभी $x \in \mathbb{R}$ के लिए
(C) $f(x)$ संतत और अवकलनीय है, सभी $x \in \mathbb{R} - \{0\}$ के लिए
(D) $f(x)$ अनंत बिंदुओं पर असंतत है
4. माना $f(x)$ अन्तराल $[a, b]$ में एक संतत फलन है और अन्तराल (a, b) में अवकलनीय है । तो यह फलन $f(x)$ अन्तराल (a, b) में निरंतर वर्धमान होगा, यदि :
- (A) $f'(x) < 0$, सभी $x \in (a, b)$ के लिए
(B) $f'(x) > 0$, सभी $x \in (a, b)$ के लिए
(C) $f'(x) = 0$, सभी $x \in (a, b)$ के लिए
(D) $f(x) > 0$, सभी $x \in (a, b)$ के लिए
5. यदि $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ है, तो $\left(\frac{24}{x} + \frac{24}{y}\right)$ का मान होगा :
- (A) 7 (B) 6
(C) 8 (D) 18
6. $\int_a^b f(x) dx$ बराबर है :
- (A) $\int_a^b f(a-x) dx$ (B) $\int_a^b f(a+b-x) dx$
(C) $\int_a^b f(x-(a+b)) dx$ (D) $\int_a^b f((a-x) + (b-x)) dx$
7. माना दो मात्रक सदिशों \hat{a} और \hat{b} के बीच का कोण θ इस प्रकार है कि $\sin \theta = \frac{3}{5}$ है । तो $\hat{a} \cdot \hat{b}$ बराबर है :
- (A) $\pm \frac{3}{5}$ (B) $\pm \frac{3}{4}$
(C) $\pm \frac{4}{5}$ (D) $\pm \frac{4}{3}$



3. Which of the following statements is true for the function

$$f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases} ?$$

- (A) $f(x)$ is continuous and differentiable $\forall x \in \mathbb{R}$
- (B) $f(x)$ is continuous $\forall x \in \mathbb{R}$
- (C) $f(x)$ is continuous and differentiable $\forall x \in \mathbb{R} - \{0\}$
- (D) $f(x)$ is discontinuous at infinitely many points

4. Let $f(x)$ be a continuous function on $[a, b]$ and differentiable on (a, b) . Then, this function $f(x)$ is strictly increasing in (a, b) if

- (A) $f'(x) < 0, \forall x \in (a, b)$
- (B) $f'(x) > 0, \forall x \in (a, b)$
- (C) $f'(x) = 0, \forall x \in (a, b)$
- (D) $f(x) > 0, \forall x \in (a, b)$

5. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is :

- (A) 7
- (B) 6
- (C) 8
- (D) 18

6. $\int_a^b f(x) dx$ is equal to :

- (A) $\int_a^b f(a-x) dx$
- (B) $\int_a^b f(a+b-x) dx$
- (C) $\int_a^b f(x-(a+b)) dx$
- (D) $\int_a^b f((a-x) + (b-x)) dx$

7. Let θ be the angle between two unit vectors \hat{a} and \hat{b} such that $\sin \theta = \frac{3}{5}$.

Then, $\hat{a} \cdot \hat{b}$ is equal to :

- (A) $\pm \frac{3}{5}$
- (B) $\pm \frac{3}{4}$
- (C) $\pm \frac{4}{5}$
- (D) $\pm \frac{4}{3}$



8. अवकल समीकरण $(1 - x^2) \frac{dy}{dx} + xy = ax$, $-1 < x < 1$, का समाकलन गुणक है :

(A) $\frac{1}{x^2 - 1}$

(B) $\frac{1}{\sqrt{x^2 - 1}}$

(C) $\frac{1}{1 - x^2}$

(D) $\frac{1}{\sqrt{1 - x^2}}$

9. यदि किसी एक रेखा के दिक्-कोसाइन $\sqrt{3} k$, $\sqrt{3} k$, $\sqrt{3} k$ हैं, तो k का मान है :

(A) ± 1

(B) $\pm \sqrt{3}$

(C) ± 3

(D) $\pm \frac{1}{3}$

10. एक रैखिक प्रोग्रामन इष्टतमकारी समस्या संबंधित होती है :

(A) लघुगणकीय फलन से

(B) रैखिक फलन से

(C) द्विघातीय फलन से

(D) चरघातांकीय फलन से

11. यदि $P(A|B) = P(A'|B)$ है, तो निम्न में से कौन-सा कथन सही है ?

(A) $P(A) = P(A')$

(B) $P(A) = 2 P(B)$

(C) $P(A \cap B) = \frac{1}{2} P(B)$

(D) $P(A \cap B) = 2 P(B)$

12. $\left| \begin{array}{cc} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{array} \right|$ बराबर है :

(A) $2x^3$

(B) 2

(C) 0

(D) $2x^3 - 2$

13. x के सापेक्ष, $\sin(x^2)$ का अवकलज, $x = \sqrt{\pi}$ पर है :

(A) 1

(B) -1

(C) $-2\sqrt{\pi}$

(D) $2\sqrt{\pi}$



8. The integrating factor of the differential equation $(1 - x^2) \frac{dy}{dx} + xy = ax$,

$-1 < x < 1$, is :

(A) $\frac{1}{x^2 - 1}$

(B) $\frac{1}{\sqrt{x^2 - 1}}$

(C) $\frac{1}{1 - x^2}$

(D) $\frac{1}{\sqrt{1 - x^2}}$

9. If the direction cosines of a line are $\sqrt{3} k, \sqrt{3} k, \sqrt{3} k$, then the value of k is :

(A) ± 1

(B) $\pm \sqrt{3}$

(C) ± 3

(D) $\pm \frac{1}{3}$

10. A linear programming problem deals with the optimization of a/an :

(A) logarithmic function

(B) linear function

(C) quadratic function

(D) exponential function

11. If $P(A|B) = P(A'|B)$, then which of the following statements is true ?

(A) $P(A) = P(A')$

(B) $P(A) = 2 P(B)$

(C) $P(A \cap B) = \frac{1}{2} P(B)$

(D) $P(A \cap B) = 2 P(B)$

12. $\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$ is equal to :

(A) $2x^3$

(B) 2

(C) 0

(D) $2x^3 - 2$

13. The derivative of $\sin(x^2)$ w.r.t. x , at $x = \sqrt{\pi}$ is :

(A) 1

(B) -1

(C) $-2\sqrt{\pi}$

(D) $2\sqrt{\pi}$



14. अवकल समीकरण $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ की कोटि और घात क्रमशः हैं :

- (A) 1, 2 (B) 2, 3
(C) 2, 1 (D) 2, 6

15. सदिश, जिसका अंतिम बिंदु A (2, -3, 5) तथा प्रारंभिक बिंदु B (3, -4, 7) है, है :

- (A) $\hat{i} - \hat{j} + 2\hat{k}$ (B) $\hat{i} + \hat{j} + 2\hat{k}$
(C) $-\hat{i} - \hat{j} - 2\hat{k}$ (D) $-\hat{i} + \hat{j} - 2\hat{k}$

16. y-अक्ष से बिंदु P(a, b, c) की दूरी है :

- (A) b (B) b^2
(C) $\sqrt{a^2 + c^2}$ (D) $a^2 + c^2$

17. व्यवरोधों $x \geq 0, y \geq 0, x + y \geq 4$ से निर्धारित सुसंगत क्षेत्र के कोनीय बिंदुओं की संख्या है :

- (A) 0 (B) 1
(C) 2 (D) 3

18. यदि दो समान कोटि वाले शून्यतर वर्ग आव्यूहों A और B के लिए $(A + B)^2 = A^2 + B^2$ है, तो :

- (A) $AB = O$ (B) $AB = -BA$
(C) $BA = O$ (D) $AB = BA$

प्रश्न संख्या 19 और 20 अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए।

- (A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।
(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या नहीं करता है।
(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है।
(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है।



14. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ respectively are :
- (A) 1, 2 (B) 2, 3
(C) 2, 1 (D) 2, 6
15. The vector with terminal point A (2, -3, 5) and initial point B (3, -4, 7) is :
- (A) $\hat{i} - \hat{j} + 2\hat{k}$ (B) $\hat{i} + \hat{j} + 2\hat{k}$
(C) $-\hat{i} - \hat{j} - 2\hat{k}$ (D) $-\hat{i} + \hat{j} - 2\hat{k}$
16. The distance of point P(a, b, c) from y-axis is :
- (A) b (B) b^2
(C) $\sqrt{a^2 + c^2}$ (D) $a^2 + c^2$
17. The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is :
- (A) 0 (B) 1
(C) 2 (D) 3
18. If A and B are two non-zero square matrices of same order such that $(A + B)^2 = A^2 + B^2$, then :
- (A) $AB = O$ (B) $AB = -BA$
(C) $BA = O$ (D) $AB = BA$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.



19. अभिकथन (A) : आव्यूह $A = \begin{bmatrix} 1 & \cos \theta & 1 \\ -\cos \theta & 1 & \cos \theta \\ -1 & -\cos \theta & 1 \end{bmatrix}$, जहाँ $\theta \in [0, 2\pi]$ के लिए,

$$|A| \in [2, 4].$$

तर्क (R) : $\cos \theta \in [-1, 1], \forall \theta \in [0, 2\pi]$.

20. अभिकथन (A) : अंतरिक्ष में एक रेखा कभी भी एक साथ x, y और z अक्षों के लंबवत नहीं हो सकती है।

तर्क (R) : किसी रेखा द्वारा x, y और z अक्षों की धनात्मक दिशाओं के साथ क्रमशः α, β और γ के कोण बनाने पर $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ है।

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. (क) जाँच कीजिए कि क्या फलन $f(x) = x^2 |x|$, बिंदु $x = 0$ पर अवकलनीय है या नहीं।

अथवा

(ख) यदि $y = \sqrt{\tan \sqrt{x}}$ है, तो सिद्ध कीजिए कि $\sqrt{x} \frac{dy}{dx} = \frac{1+y^4}{4y}$.

22. दर्शाइए कि फलन $f(x) = 4x^3 - 18x^2 + 27x - 7$ का उच्चतम या निम्नतम मान नहीं है।

23. (क) ज्ञात कीजिए :

$$\int x \sqrt{1+2x} dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_0^{\frac{\pi^2}{4}} \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$



19. Assertion (A) : For matrix $A = \begin{bmatrix} 1 & \cos \theta & 1 \\ -\cos \theta & 1 & \cos \theta \\ -1 & -\cos \theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$,

$$|A| \in [2, 4].$$

Reason (R) : $\cos \theta \in [-1, 1], \forall \theta \in [0, 2\pi]$.

20. Assertion (A) : A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.

Reason (R) : For any line making angles, α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Check whether the function $f(x) = x^2 |x|$ is differentiable at $x = 0$ or not.

OR

(b) If $y = \sqrt{\tan \sqrt{x}}$, prove that $\sqrt{x} \frac{dy}{dx} = \frac{1+y^4}{4y}$.

22. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

23. (a) Find :

$$\int x \sqrt{1+2x} \, dx$$

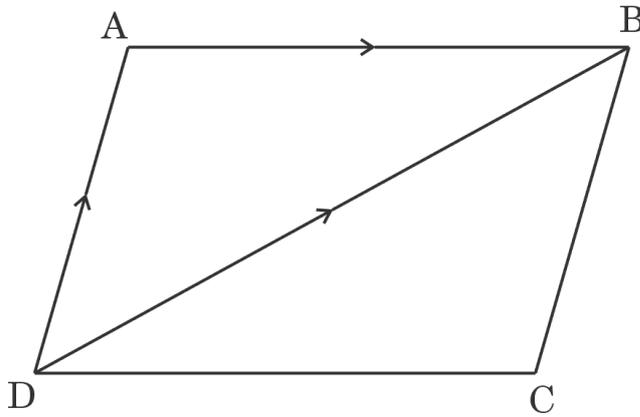
OR

(b) Evaluate :

$$\int_0^{\frac{\pi^2}{4}} \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx$$



24. यदि दो शून्येतर सदिश \vec{a} और \vec{b} इस प्रकार हैं कि $(\vec{a} + \vec{b}) \perp \vec{a}$ और $(2\vec{a} + \vec{b}) \perp \vec{b}$, तो सिद्ध कीजिए कि $|\vec{b}| = \sqrt{2} |\vec{a}|$.
25. दी गई आकृति में, ABCD एक समांतर चतुर्भुज है। यदि $\vec{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ तथा $\vec{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ हैं, तो \vec{AD} ज्ञात कीजिए और इसके प्रयोग से समांतर चतुर्भुज ABCD का क्षेत्रफल ज्ञात कीजिए।



खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. (क) समुच्चय $A = \{1, 2, 3, 4, 5\}$ में एक संबंध $R = \{(x, y) : |x^2 - y^2| < 8\}$ द्वारा परिभाषित है। जाँच कीजिए कि क्या यह संबंध R स्वतुल्य, सममित और संक्रामक है।

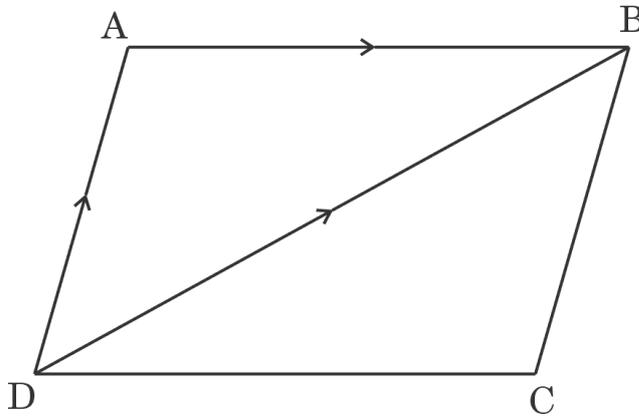
अथवा

- (ख) फलन $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = ax + b$ द्वारा इस प्रकार परिभाषित है कि $f(1) = 1$ और $f(2) = 3$. फलन $f(x)$ ज्ञात कीजिए। अतः, जाँच कीजिए कि क्या फलन $f(x)$ एकैकी और आच्छादक है या नहीं।



24. If \vec{a} and \vec{b} are two non-zero vectors such that $(\vec{a} + \vec{b}) \perp \vec{a}$ and $(2\vec{a} + \vec{b}) \perp \vec{b}$, then prove that $|\vec{b}| = \sqrt{2} |\vec{a}|$.

25. In the given figure, ABCD is a parallelogram. If $\vec{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find \vec{AD} and hence find the area of parallelogram ABCD.



SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) A relation R on set $A = \{1, 2, 3, 4, 5\}$ is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.

OR

(b) A function f is defined from $\mathbb{R} \rightarrow \mathbb{R}$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.



27. (क) यदि $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ है, तो सिद्ध कीजिए कि

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

अथवा

(ख) यदि $y = (\tan x)^x$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए ।

28. (क) ज्ञात कीजिए :

$$\int \frac{x^2}{(x^2+4)(x^2+9)} dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_1^3 (|x-1| + |x-2| + |x-3|) dx$$

29. अवकल समीकरण $x^2 \frac{dy}{dx} - xy = x^2 \cos^2 \left(\frac{y}{2x} \right)$ का विशिष्ट हल ज्ञात कीजिए, दिया

हुआ है कि $y = \frac{\pi}{2}$, यदि $x = 1$.

30. निम्न रैखिक प्रोग्रामन समस्या को आलेखीय विधि द्वारा हल कीजिए :

निम्न व्यवरोधों के अंतर्गत

$$x + 2y \leq 12$$

$$2x + y \leq 12$$

$$4x + 5y \geq 20$$

$$x \geq 0, y \geq 0$$

$z = 500x + 300y$ का अधिकतमीकरण कीजिए ।

31. E और F दो स्वतंत्र घटनाएँ ऐसी हैं जिनके लिए $P(\bar{E}) = 0.6$ तथा $P(E \cup F) = 0.6$ है । $P(F)$ और $P(\bar{E} \cup \bar{F})$ ज्ञात कीजिए ।



27. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(b) If $y = (\tan x)^x$, then find $\frac{dy}{dx}$.

28. (a) Find :

$$\int \frac{x^2}{(x^2+4)(x^2+9)} dx$$

OR

(b) Evaluate :

$$\int_1^3 (|x-1| + |x-2| + |x-3|) dx$$

29. Find the particular solution of the differential equation given by $x^2 \frac{dy}{dx} - xy = x^2 \cos^2\left(\frac{y}{2x}\right)$, given that when $x = 1$, $y = \frac{\pi}{2}$.

30. Solve the following linear programming problem graphically :

Maximise $z = 500x + 300y$,

subject to constraints

$$x + 2y \leq 12$$

$$2x + y \leq 12$$

$$4x + 5y \geq 20$$

$$x \geq 0, y \geq 0$$

31. E and F are two independent events such that $P(\bar{E}) = 0.6$ and $P(E \cup F) = 0.6$. Find $P(F)$ and $P(\bar{E} \cup \bar{F})$.

खण्ड घ

इस खण्ड में दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. (क) यदि $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए और इसके प्रयोग से, निम्न

समीकरण निकाय को हल कीजिए :

$$x - 2y = 10, 2x - y - z = 8, -2y + z = 7$$

अथवा

(ख) यदि $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ तथा $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$ है,

तो $(a + x) - (b + y)$ का मान ज्ञात कीजिए।

33. (क) मान ज्ञात कीजिए :

$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$$

34. समाकलन विधि के प्रयोग से, दीर्घवृत्त $\frac{x^2}{16} + \frac{y^2}{4} = 1$ के उस क्षेत्र का, जो रेखाओं $x = -2$ और $x = 2$ के बीच है, क्षेत्रफल ज्ञात कीजिए।

35. यदि रेखा $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ में बिंदु $P(x, y, z)$ का प्रतिबिम्ब $P'(1, 0, 7)$ है, तो बिंदु P के निर्देशांक ज्ञात कीजिए।

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. (a) If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following

system of equations :

$$x - 2y = 10, 2x - y - z = 8, -2y + z = 7$$

OR

(b) If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$,

find the value of $(a + x) - (b + y)$.

33. (a) Evaluate :

$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

OR

(b) Evaluate :

$$\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$$

34. Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines $x = -2$ and $x = 2$.

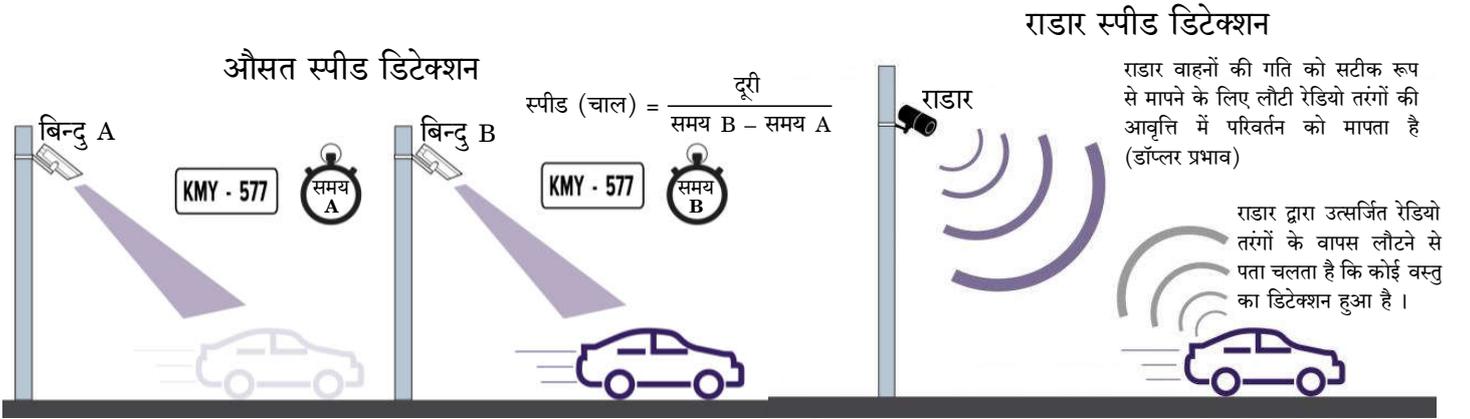
35. The image of point $P(x, y, z)$ with respect to line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is $P'(1, 0, 7)$. Find the coordinates of point P .

खण्ड ड

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36. ट्रैफिक पुलिस ने शहर में विभिन्न स्थानों पर ओवर स्पीड उल्लंघन डिटेक्शन (OSVD) प्रणाली स्थापित की है। ये कैमरे 300 मीटर की दूरी से तेज गति से चलने वाले वाहन की फोटो ले सकते हैं और अँधेरे में भी काम कर सकते हैं।



एक खंभे पर 5 मीटर की ऊँचाई पर एक कैमरा स्थापित किया गया है। यह 20 मीटर / सेकंड की गति से खंभे से दूर जा रही एक कार का पता लगाता है। खंभे के पाद से x मीटर दूरी पर किसी भी बिंदु पर, कार C से स्पीड कैमरे का उन्नयन कोण θ है।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

- (i) खंभे पर स्थापित किए गए कैमरे की ऊँचाई और x के रूप में θ को व्यक्त कीजिए। 1
- (ii) $\frac{d\theta}{dx}$ ज्ञात कीजिए। 1
- (iii) (क) जब कार खंभे से 50 मीटर दूर हो, तो उस क्षण पर समय के सापेक्ष उन्नयन कोण में परिवर्तन की दर ज्ञात कीजिए। 2

अथवा

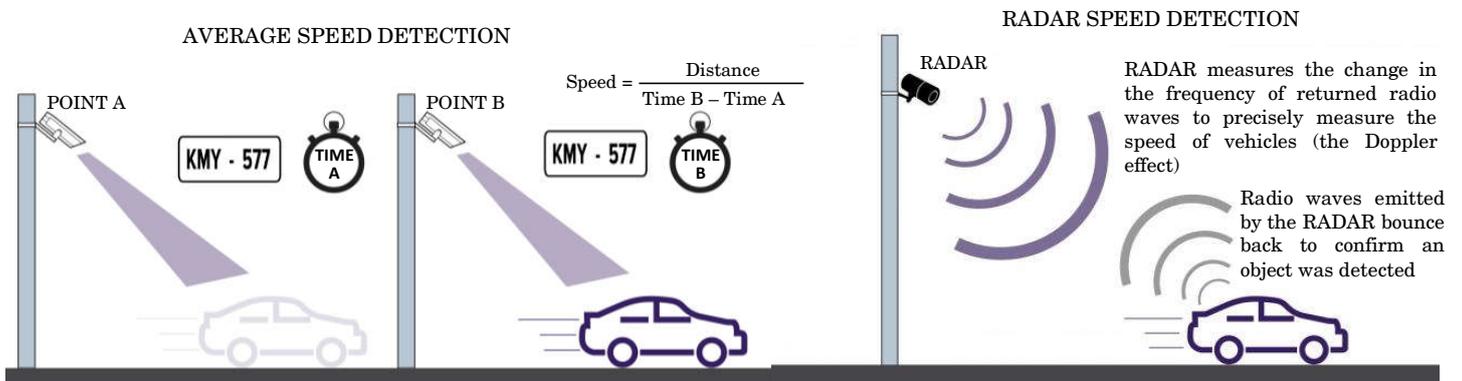
- (iii) (ख) यदि खंभे के पाद से 50 मीटर की दूरी पर दूसरी कार के समय के सापेक्ष उन्नयन कोण में परिवर्तन की दर $\frac{3}{101}$ रेडियन/सेकंड है, तो कार की गति ज्ञात कीजिए। 2

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

- 36.** The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ .

On the basis of the above information, answer the following questions :

- (i) Express θ in terms of height of the camera installed on the pole and x . 1
- (ii) Find $\frac{d\theta}{dx}$. 1
- (iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. 2

OR

- (iii) (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car. 2

प्रकरण अध्ययन - 2

37. हाल के शोध के अनुसार, जलवायु परिवर्तन के कारण दुनिया भर के विभिन्न क्षेत्रों में वायु विकोभ बढ़ता है। वायु विकोभ उड़ान को मुश्किल बना देता है और अक्सर उड़ान में देरी करता है।

मान लीजिए कि एक हवाई जहाज समान प्रायिकता के साथ गंभीर विकोभ, मध्यम विकोभ या हल्के विकोभ का अनुभव करता है। इसके अलावा, गंभीर विकोभ, मध्यम विकोभ और हल्के विकोभ के कारण हवाई जहाज के गंतव्य पर देर से पहुँचने की प्रायिकता क्रमशः 55%, 37% और 17% है।



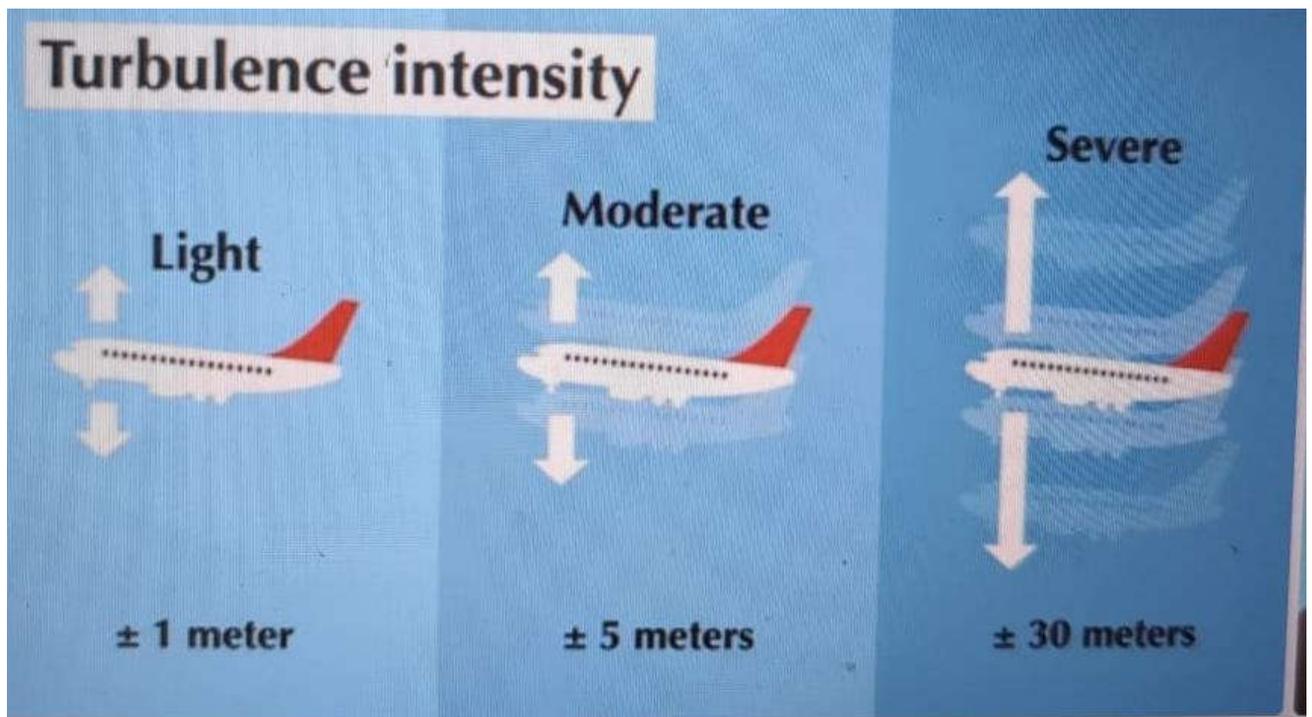
उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

- (i) हवाई जहाज के गंतव्य पर देर से पहुँचने की प्रायिकता ज्ञात कीजिए। 2
- (ii) यदि हवाई जहाज अपने गंतव्य पर देर से पहुँचता है, तो प्रायिकता ज्ञात कीजिए कि ऐसा मध्यम विकोभ के कारण हुआ है। 2

Case Study – 2

37. According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.

Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.

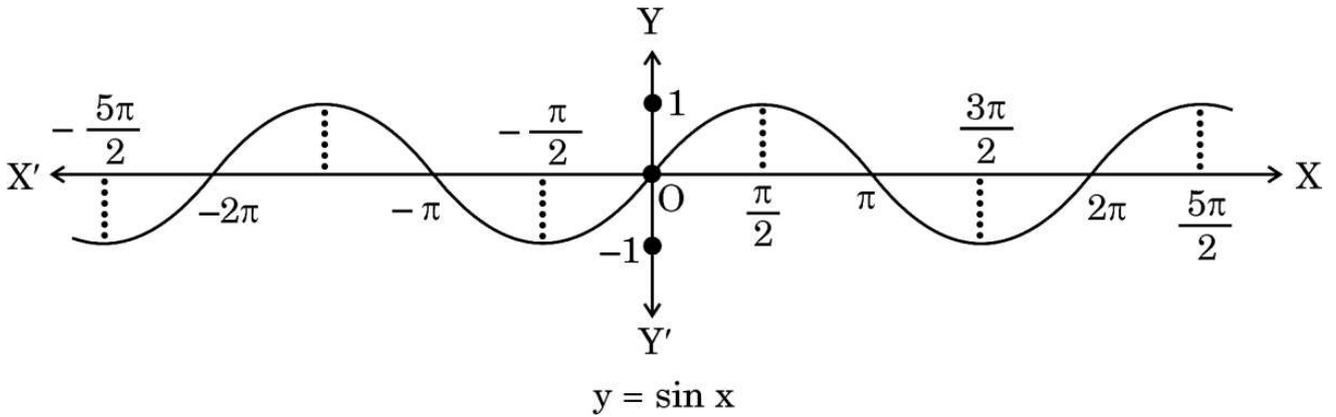


On the basis of the above information, answer the following questions :

- (i) Find the probability that an airplane reached its destination late. 2
- (ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence. 2

प्रकरण अध्ययन - 3

38. यदि फलन $f : X \rightarrow Y$ इस प्रकार परिभाषित है कि $f(x) = y$ एकैकी तथा आच्छादक हो, तो हम एक अद्वितीय फलन $g : Y \rightarrow X$ इस प्रकार परिभाषित कर सकते हैं कि $g(y) = x$, जहाँ $x \in X$ तथा $y = f(x)$, $y \in Y$ है। फलन g को फलन f का प्रतिलोम कहा जाता है।
- sine फलन का प्रांत \mathbb{R} और फलन $\text{sine} : \mathbb{R} \rightarrow \mathbb{R}$ न तो एकैकी है और न ही आच्छादक है। निम्न आकृति में sine फलन का आलेख दिखाया गया है।



मान लीजिए sine फलन समुच्चय A से $[-1, 1]$ इस प्रकार परिभाषित है कि sine फलन के प्रतिलोम का अस्तित्व है, यानि $\sin^{-1} x : [-1, 1] \rightarrow A$ पर परिभाषित है।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

- (i) यदि A मुख्य मान शाखा के अलावा अन्य अंतराल है, तो ऐसे एक अंतराल का उदाहरण दीजिए। 1
- (ii) यदि $\sin^{-1}(x)$ को $[-1, 1]$ से इसकी मुख्य मान शाखा में परिभाषित किया गया हो, तो $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$ का मान ज्ञात कीजिए। 1
- (iii) (क) $[-1, 1]$ से मुख्य मान शाखा तक के लिए $\sin^{-1} x$ का आलेख बनाइए। 2

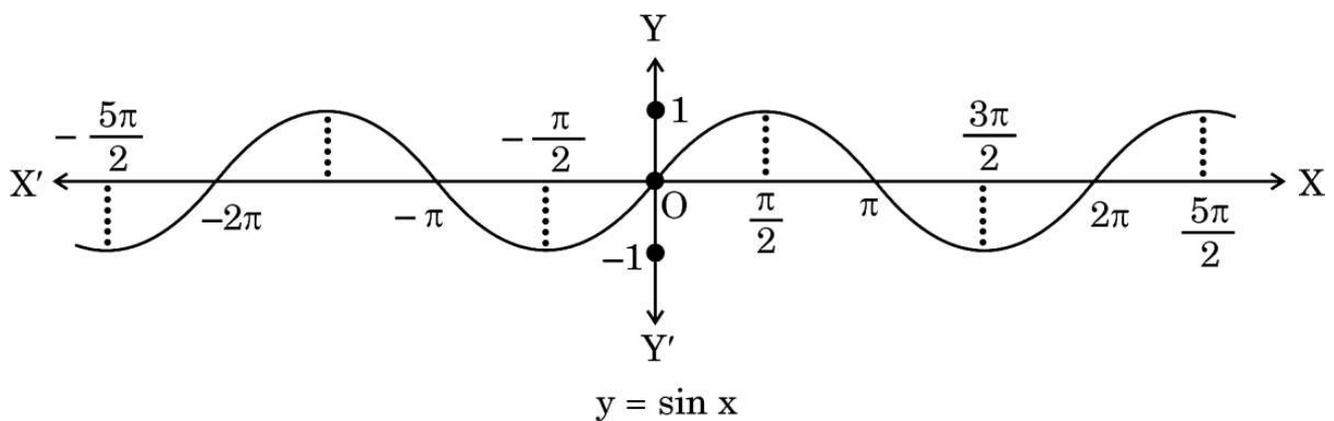
अथवा

- (iii) (ख) $f(x) = 2 \sin^{-1}(1 - x)$ का प्रांत और परिसर ज्ञात कीजिए। 2

Case Study – 3

38. If a function $f : X \rightarrow Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \rightarrow X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$, $y \in Y$. Function g is called the inverse of function f .

The domain of sine function is \mathbb{R} and function $\text{sine} : \mathbb{R} \rightarrow \mathbb{R}$ is neither one-one nor onto. The following graph shows the sine function.



Let sine function be defined from set A to $[-1, 1]$ such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from $[-1, 1]$ to A .

On the basis of the above information, answer the following questions :

- (i) If A is the interval other than principal value branch, give an example of one such interval. 1
- (ii) If $\sin^{-1}(x)$ is defined from $[-1, 1]$ to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. 1
- (iii) (a) Draw the graph of $\sin^{-1} x$ from $[-1, 1]$ to its principal value branch. 2

OR

- (iii) (b) Find the domain and range of $f(x) = 2 \sin^{-1}(1 - x)$. 2

Marking Scheme
Strictly Confidential
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Senior School Certificate Examination, 2024
MATHEMATICS PAPER CODE - 65/1/1

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC.”
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark(\surd) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (\surd)while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note “Extra Question”.</u>

10	<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question”.</u>
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	<p>Ensure that you do not make the following common types of errors committed by the Examiner in the past:-</p> <ul style="list-style-type: none"> ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) <p>Half or a part of answer marked correct and the rest as wrong, but no marks awarded.</p>
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the “ Guidelines for spot Evaluation ” before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code-041)
(PAPER CODE: 65/1/1)

Section A

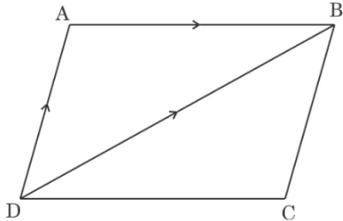
Q.No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each .	
1.	A function $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ (where \mathbb{R}_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is : (A) one-one but not onto (B) onto but not one-one (C) both one-one and onto (D) neither one-one nor onto	
Sol.	(A) one-one but not onto	1
2.	If a matrix has 36 elements, the number of possible orders it can have, is : (A) 13 (B) 3 (C) 5 (D) 9	
Sol.	(D) 9	1
3.	Which of the following statements is true for the function $f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases} ?$ (A) $f(x)$ is continuous and differentiable $\forall x \in \mathbb{R}$ (B) $f(x)$ is continuous $\forall x \in \mathbb{R}$ (C) $f(x)$ is continuous and differentiable $\forall x \in \mathbb{R} - \{0\}$ (D) $f(x)$ is discontinuous at infinitely many points	
Sol.	(C) $f(x)$ is continuous and differentiable $\forall x \in \mathbb{R} - \{0\}$	1

4.	Let $f(x)$ be a continuous function on $[a, b]$ and differentiable on (a, b) . Then, this function $f(x)$ is strictly increasing in (a, b) if (A) $f'(x) < 0, \forall x \in (a, b)$ (B) $f'(x) > 0, \forall x \in (a, b)$ (C) $f'(x) = 0, \forall x \in (a, b)$ (D) $f(x) > 0, \forall x \in (a, b)$	
Sol.	(B) $f'(x) > 0, \forall x \in (a, b)$	1
5.	If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is : (A) 7 (B) 6 (C) 8 (D) 18	
Sol.	(D) 18	1
6.	$\int_a^b f(x) dx$ is equal to : (A) $\int_a^b f(a-x) dx$ (B) $\int_a^b f(a+b-x) dx$ (C) $\int_a^b f(x-(a+b)) dx$ (D) $\int_a^b f((a-x) + (b-x)) dx$	
Sol.	(B) $\int_a^b f(a+b-x) dx$	1
7.	Let θ be the angle between two unit vectors \hat{a} and \hat{b} such that $\sin \theta = \frac{3}{5}$. Then, $\hat{a} \cdot \hat{b}$ is equal to : (A) $\pm \frac{3}{5}$ (B) $\pm \frac{3}{4}$ (C) $\pm \frac{4}{5}$ (D) $\pm \frac{4}{3}$	
Sol.	(C) $\pm \frac{4}{5}$	1
8.	The integrating factor of the differential equation $(1-x^2) \frac{dy}{dx} + xy = ax$, $-1 < x < 1$, is : (A) $\frac{1}{x^2-1}$ (B) $\frac{1}{\sqrt{x^2-1}}$ (C) $\frac{1}{1-x^2}$ (D) $\frac{1}{\sqrt{1-x^2}}$	
Sol.	(D) $\frac{1}{\sqrt{1-x^2}}$	1

9.	If the direction cosines of a line are $\sqrt{3}k, \sqrt{3}k, \sqrt{3}k$, then the value of k is : (A) ± 1 (B) $\pm \sqrt{3}$ (C) ± 3 (D) $\pm \frac{1}{3}$	
Sol.	(D) $\pm \frac{1}{3}$	1
10.	A linear programming problem deals with the optimization of a/an : (A) logarithmic function (B) linear function (C) quadratic function (D) exponential function	
Sol.	(B) linear function	1
11.	If $P(A B) = P(A' B)$, then which of the following statements is true ? (A) $P(A) = P(A')$ (B) $P(A) = 2P(B)$ (C) $P(A \cap B) = \frac{1}{2}P(B)$ (D) $P(A \cap B) = 2P(B)$	
Sol.	(C) $P(A \cap B) = \frac{1}{2}P(B)$	1
12.	$\left \begin{matrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{matrix} \right $ is equal to : (A) $2x^3$ (B) 2 (C) 0 (D) $2x^3 - 2$	
Sol.	(B) 2	1
13.	The derivative of $\sin(x^2)$ w.r.t. x , at $x = \sqrt{\pi}$ is : (A) 1 (B) -1 (C) $-2\sqrt{\pi}$ (D) $2\sqrt{\pi}$	
Sol.	(C) $-2\sqrt{\pi}$	1
14.	The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ respectively are : (A) 1, 2 (B) 2, 3 (C) 2, 1 (D) 2, 6	
Sol.	(C) 2, 1	1

15.	The vector with terminal point A (2, - 3, 5) and initial point B (3, - 4, 7) is : (A) $\hat{i} - \hat{j} + 2\hat{k}$ (B) $\hat{i} + \hat{j} + 2\hat{k}$ (C) $-\hat{i} - \hat{j} - 2\hat{k}$ (D) $-\hat{i} + \hat{j} - 2\hat{k}$	
Sol.	(D) $-\hat{i} + \hat{j} - 2\hat{k}$	1
16.	The distance of point P(a, b, c) from y-axis is : (A) b (B) b^2 (C) $\sqrt{a^2 + c^2}$ (D) $a^2 + c^2$	
Sol.	(C) $\sqrt{a^2 + c^2}$	1
17.	The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is : (A) 0 (B) 1 (C) 2 (D) 3	
Sol.	(C) 2	1
18.	If A and B are two non-zero square matrices of same order such that $(A + B)^2 = A^2 + B^2$, then : (A) $AB = O$ (B) $AB = -BA$ (C) $BA = O$ (D) $AB = BA$	
Sol.	(B) $AB = -BA$	1
	Questions No. 19 & 20, are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below : (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	Assertion (A) : For matrix $A = \begin{bmatrix} 1 & \cos \theta & 1 \\ -\cos \theta & 1 & \cos \theta \\ -1 & -\cos \theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$, $ A \in [2, 4]$. Reason (R) : $\cos \theta \in [-1, 1], \forall \theta \in [0, 2\pi]$.	
Sol.	(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).	1

23(a).	Find : $\int x \sqrt{1+2x} \, dx$	
Sol.	$1 + 2x = t^2$ $2 \, dx = 2t \, dt$ $\frac{1}{2} \int (t^4 - t^2) \, dt = \frac{1}{2} \left[\frac{t^5}{5} - \frac{t^3}{3} \right] + C$ $= \frac{(1+2x)^{\frac{5}{2}}}{10} - \frac{(1+2x)^{\frac{3}{2}}}{6} + C$	$\frac{1}{2}$ 1 $\frac{1}{2}$
OR		
23(b).	Evaluate : $\int_0^{\frac{\pi^2}{4}} \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx$	
Sol.	$\int_0^{\frac{\pi^2}{4}} \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx$ $2 \int_0^{\frac{\pi}{2}} \sin t \, dt = 2 [-\cos t]_0^{\frac{\pi}{2}}$ $= 2$ <p style="text-align: right;">Put $\sqrt{x} = t \Rightarrow dx = 2t \, dt$</p>	$\frac{1}{2}$ 1 $\frac{1}{2}$
24.	If \vec{a} and \vec{b} are two non-zero vectors such that $(\vec{a} + \vec{b}) \perp \vec{a}$ and $(2\vec{a} + \vec{b}) \perp \vec{b}$, then prove that $ \vec{b} = \sqrt{2} \vec{a} $.	
Sol.	$(\vec{a} + \vec{b}) \cdot \vec{a} = 0 \Rightarrow \vec{a} ^2 + \vec{b} \cdot \vec{a} = 0 \text{ -----(1)}$ $(2\vec{a} + \vec{b}) \cdot \vec{b} = 0 \Rightarrow 2\vec{a} \cdot \vec{b} + \vec{b} ^2 = 0 \text{ -----(2)}$ $2(- \vec{a} ^2) + \vec{b} ^2 = 0 \text{ {Using (1) and (2)}}$ $ \vec{b} ^2 = 2 \vec{a} ^2 \Rightarrow \vec{b} = \sqrt{2} \vec{a} $	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

25.	<p>In the given figure, ABCD is a parallelogram. If $\vec{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find \vec{AD} and hence find the area of parallelogram ABCD.</p> 	
Sol.	$\vec{AD} + \vec{DB} = \vec{AB}$ $\vec{AD} = (2\hat{i} - 4\hat{j} + 5\hat{k}) - (3\hat{i} - 6\hat{j} + 2\hat{k})$ $= -\hat{i} + 2\hat{j} + 3\hat{k}$ $\vec{AD} \times \vec{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ 2 & -4 & 5 \end{vmatrix} = 22\hat{i} + 11\hat{j}$ $\text{Area} = \vec{AD} \times \vec{AB} = 22\hat{i} + 11\hat{j} $ $= \sqrt{605} \text{ or } 11\sqrt{5}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
SECTION C In this section there are 6 short answer type questions of 3 marks each.		
26(a).	<p>A relation R on set $A = \{1, 2, 3, 4, 5\}$ is defined as $R = \{(x, y) : x^2 - y^2 < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.</p>	
Sol.	<p>(a) Reflexive: $\because x^2 - x^2 < 8 \forall x \in A \Rightarrow (x, x) \in R \therefore R$ is reflexive.</p> <p>(b) Symmetric: Let $(x, y) \in R$ for some $x, y \in A$ $\therefore x^2 - y^2 < 8 \Rightarrow y^2 - x^2 < 8 \Rightarrow (y, x) \in R$ Hence R is symmetric.</p> <p>(c) Transitive: $(1, 2), (2, 3) \in R$ as $1^2 - 2^2 < 8, 2^2 - 3^2 < 8$ respectively But $1^2 - 3^2 \not< 8 \Rightarrow (1, 3) \notin R$ Hence R is not transitive.</p>	$\frac{1}{2}$ 1 $1\frac{1}{2}$
OR		
26(b).	<p>A function f is defined from $R \rightarrow R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function $f(x)$. Hence, check whether function $f(x)$ is one-one and onto or not.</p>	

Sol.	<p>$f(x) = ax + b$</p> <p>Solving $a + b = 1$ and $2a + b = 3$ to get $a=2, b = -1$</p> <p>$f(x) = 2x - 1$</p> <p>Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$</p> <p>$2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2$</p> <p>Hence f is one - one.</p> <p>Let $y = 2x - 1, y \in R$ (Codomain)</p> <p>$\Rightarrow x = \frac{y+1}{2} \in R$ (domain)</p> <p>Also, $f(x) = f\left(\frac{y+1}{2}\right) = y$</p> <p>$\therefore f$ is onto.</p>	<p>1</p> <p>1</p> <p>1</p>
27(a).	<p>If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.</p>	
Sol.	<p>$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$</p> <p>Put $x = \sin \theta, y = \sin \phi$</p> <p>$\Rightarrow \cos \theta + \cos \phi = a(\sin \theta - \sin \phi)$</p> <p>$\Rightarrow 2 \cos\left(\frac{\theta+\phi}{2}\right) \cos\left(\frac{\theta-\phi}{2}\right) = 2a \sin\left(\frac{\theta-\phi}{2}\right) \cos\left(\frac{\theta+\phi}{2}\right)$</p> <p>$\Rightarrow \cot\left(\frac{\theta-\phi}{2}\right) = a$</p> <p>$\Rightarrow \theta - \phi = 2 \cot^{-1} a$</p> <p>$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2 \cot^{-1} a$</p> <p>$\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0$</p> <p>$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p>
OR		
27(b).	<p>If $y = (\tan x)^x$, then find $\frac{dy}{dx}$.</p>	
Sol.	<p>$y = (\tan x)^x$</p> <p>$\log y = x \log(\tan x)$</p> <p>$\frac{1}{y} \frac{dy}{dx} = x \left(\frac{\sec^2 x}{\tan x}\right) + \log(\tan x)$</p> <p>$\frac{dy}{dx} = (\tan x)^x \left[\left(\frac{x \sec^2 x}{\tan x}\right) + \log(\tan x)\right]$</p>	<p>$\frac{1}{2}$</p> <p>2</p> <p>$\frac{1}{2}$</p>
28(a).	<p>Find :</p> $\int \frac{x^2}{(x^2 + 4)(x^2 + 9)} dx$	

Sol.	<p>Let $I = \int \frac{x^2}{(x^2+4)(x^2+9)} dx$</p> <p>Put $x^2 = t$</p> $\frac{t}{(t+4)(t+9)} = \frac{A}{t+4} + \frac{B}{t+9} \Rightarrow A = \frac{-4}{5}, B = \frac{9}{5}$ $I = \frac{-4}{5} \int \frac{1}{2^2+x^2} dx + \frac{9}{5} \int \frac{1}{3^2+x^2} dx$ $= \frac{-2}{5} \tan^{-1}\left(\frac{x}{2}\right) + \frac{3}{5} \tan^{-1}\left(\frac{x}{3}\right) + C$	$\frac{1}{2}$ $\frac{1}{2}$ 1
OR		
28(b).	<p>Evaluate :</p> $\int_1^3 (x-1 + x-2 + x-3) dx$	
Sol.	$\int_1^3 (x-1 + x-2 + x-3) dx$ $= \int_1^2 (x-1) dx + \int_2^3 -(x-2) dx + \int_2^3 (x-2) dx - \int_1^3 (x-3) dx$ $= \int_1^2 2 dx + \int_1^2 (2-x) dx + \int_2^3 (x-2) dx$ $= [2x]_1^2 + \left[\frac{(2-x)^2}{-2}\right]_1^2 + \left[\frac{(x-2)^2}{2}\right]_2^3$ $= 4 + \frac{1}{2} + \frac{1}{2} = 5$	$\frac{1}{2}$ $\frac{1}{2}$ 1
29.	<p>Find the particular solution of the differential equation given by</p> $x^2 \frac{dy}{dx} - xy = x^2 \cos^2\left(\frac{y}{2x}\right), \text{ given that when } x = 1, y = \frac{\pi}{2}.$	
Sol.	$\frac{dy}{dx} = \frac{y}{x} + \cos^2\left(\frac{y}{2x}\right)$ <p>Put $y = vx$ so that $\frac{dy}{dx} = v + x \frac{dv}{dx}$</p> $\Rightarrow v + x \frac{dv}{dx} = v + \cos^2\left(\frac{v}{2}\right)$ $\Rightarrow \int \sec^2\left(\frac{v}{2}\right) dv = \int \frac{1}{x} dx$ $\Rightarrow 2 \tan\left(\frac{v}{2}\right) = \log x + C$ $\Rightarrow 2 \tan\left(\frac{y}{2x}\right) = \log x + C$ $2 \tan\left(\frac{\pi}{4}\right) = \log 1 + C \Rightarrow C = 2 \Rightarrow 2 \tan\left(\frac{y}{2x}\right) = \log x + 2$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ 1

30. Solve the following linear programming problem graphically :
 Maximise $z = 500x + 300y$,
 subject to constraints

$$x + 2y \leq 12$$

$$2x + y \leq 12$$

$$4x + 5y \geq 20$$

$$x \geq 0, y \geq 0$$

Sol. $\text{Max } z = 500x + 300y$

Corner Point	Z
A (0,4)	1200
B (0,6)	1800
C (4,4)	3200
D (6,0)	3000
E (5,0)	2500

$\text{Max } z = 3200 \text{ at } x = 4, y = 4$

Correct
Graph -
 $1\frac{1}{2}$

Correct
Table -
1

$\frac{1}{2}$

31. \bar{E} and \bar{F} are two independent events such that $P(\bar{E}) = 0.6$ and $P(E \cup F) = 0.6$. Find $P(F)$ and $P(\bar{E} \cup \bar{F})$.

Sol. $P(\bar{E}) = 0.6 \Rightarrow P(E) = 0.4$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$\Rightarrow 0.6 = 0.4 + P(F) - 0.4 P(F) \Rightarrow P(F) = \frac{1}{3}$$

$$P(\bar{E} \cup \bar{F}) = 1 - P(E \cap F)$$

$$= 1 - 0.4 \times \frac{1}{3} = \frac{13}{15}$$

$\frac{1}{2}$

$\frac{1}{2}$

1

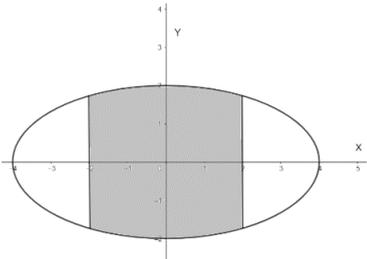
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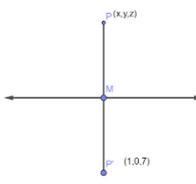
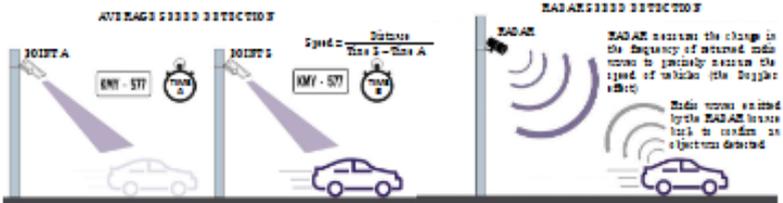
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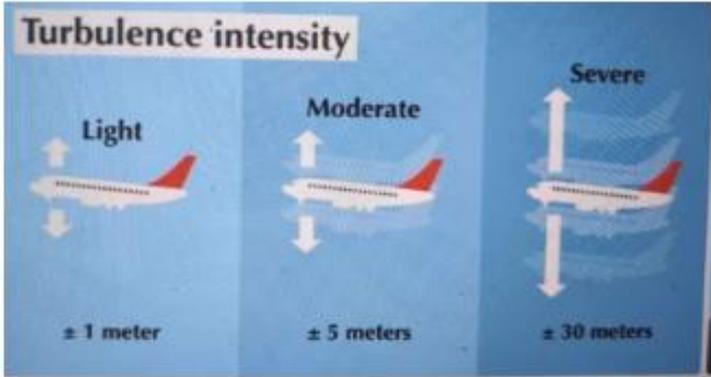
SECTION D

In this section there are 4 long answer type questions of 5 marks each.

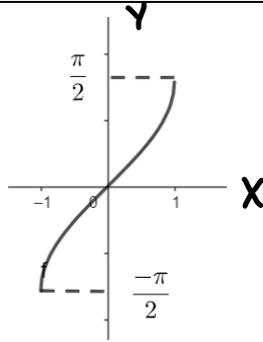
32(a).	<p>If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :</p> $x - 2y = 10, 2x - y - z = 8, -2y + z = 7$	
Sol.	<p>$A = 1 \neq 0$ hence A^{-1} exists.</p> $\text{Adj } A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$ $AX = B \Rightarrow \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$ $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ <p>$\Rightarrow x = 0, y = -5, z = -3$</p>	<p>1 2 $\frac{1}{2}$ $\frac{1}{2}$</p>
OR		
32(b).	<p>If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$,</p> <p>find the value of $(a + x) - (b + y)$.</p>	
Sol.	<p>$AA^{-1} = I$</p> $\begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} -1 - 8a + 2b & 1 + 7a + 2y & 5 - 5a \\ -15 + bx & 13 + xy & 3x - 9 \\ -5 + b & 4 + y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ <p>$-5 + b = 0 \Rightarrow b = 5, \quad 5 - 5a = 0 \Rightarrow a = 1$</p> <p>$4 + y = 0 \Rightarrow y = -4, \quad 3x - 9 = 0 \Rightarrow x = 3$</p> <p>$\therefore (a + x) - (b + y) = (1 + 3) - (5 - 4) = 3$</p>	<p>1 $\frac{1}{2}$ 1 1 $\frac{1}{2}$</p>
33(a).	<p>Evaluate :</p> $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$	
Sol.	<p>Let $I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$</p>	

	<p>Put $\sin x - \cos x = t$, so that $(\cos x + \sin x) dx = dt$</p> $\sin^2 x + \cos^2 x - \sin 2x = t^2 \Rightarrow \sin 2x = 1 - t^2$ $I = \int_{-1}^0 \frac{dt}{25-16t^2}$ $= \frac{1}{16} \int_{-1}^0 \frac{dt}{(\frac{5}{4})^2 - t^2}$ $= \frac{1}{40} \left[\log \left \frac{5+4t}{5-4t} \right \right]_{-1}^0$ $= \frac{1}{40} \left[\log 1 - \log \left(\frac{1}{9} \right) \right] = \frac{1}{40} \log 9 \text{ or } \frac{1}{20} \log 3$	<p>1</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$1\frac{1}{2}$</p> <p>1</p>
	OR	
33(b).	<p>Evaluate :</p> $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$	
Sol.	<p>Let $I = \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$</p> <p>Put $\sin x = t$ so that $\cos x dx = dt$</p> $I = 2 \int_0^1 t \tan^{-1} t dt$ $= 2 \left[\tan^{-1} t \left(\frac{t^2}{2} \right) - \frac{1}{2} \int \frac{t^2}{1+t^2} dt \right]_0^1$ $= 2 \left[\left(\frac{t^2}{2} \right) \tan^{-1} t - \frac{1}{2} t + \frac{1}{2} \tan^{-1} t \right]_0^1$ $= 2 \left(\frac{\pi}{4} - \frac{1}{2} \right) = \frac{\pi}{2} - 1$	<p>1</p> <p>1</p> <p>$1\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p>
34.	<p>Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines $x = -2$ and $x = 2$.</p>	
Sol.	 <p>Area = $4 \int_0^2 y dx$</p> $= 4 \left[\frac{1}{2} \int_0^2 \sqrt{4^2 - x^2} dx \right]$ $= 2 \left[\frac{x}{2} \sqrt{4^2 - x^2} + 8 \sin^{-1} \left(\frac{x}{4} \right) \right]_0^2$ $= 2 \left[\sqrt{12} + \frac{8\pi}{6} \right] = 4\sqrt{3} + \frac{8\pi}{3}$	<p>Correct graph-1</p> <p>1</p> <p>2</p> <p>1</p>

35.	<p>The image of point $P(x, y, z)$ with respect to line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is $P'(1, 0, 7)$. Find the coordinates of point P.</p>	
Sol.	<p>Let foot of the perpendicular on the given line from point P be $M(\lambda, 2\lambda + 1, 3\lambda + 2)$</p> <p>D. ratios of PP' are $\lambda - 1, 2\lambda + 1, 3\lambda - 5$</p> $1(\lambda - 1) + 2(2\lambda + 1) + 3(3\lambda - 5) = 0$ $\Rightarrow \lambda = 1$ <p>Coordinates of $M(1, 3, 5)$</p> $\frac{x+1}{2} = 1, \frac{y+0}{2} = 3, \frac{z+7}{2} = 5$ $\Rightarrow x = 1, y = 6, z = 3 \Rightarrow P(1, 6, 3)$ 	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
<p>SECTION E</p> <p>In this section there are 3 case-study based questions of 4 marks each.</p>		
36.	<p>The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.</p>  <p>A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.</p> <p>On the basis of the above information, answer the following questions :</p> <p>(i) Express θ in terms of height of the camera installed on the pole and x. 1</p> <p>(ii) Find $\frac{d\theta}{dx}$. 1</p> <p>(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. 2</p> <p style="text-align: center;">OR</p> <p>(iii) (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car. 2</p>	
Sol.	<p>(i) $\tan \theta = \frac{5}{x} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{x}\right)$</p> <p>(ii) $\frac{d\theta}{dx} = \frac{-5}{5^2 + x^2}$</p>	<p>1</p> <p>1</p>

	<p>(iii) (a) $\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt} = \frac{-5}{5^2+x^2} \times 20 \Big _{x=50}$</p> $= \frac{-100}{2525} \text{ or } \frac{-4}{101} \text{ rad/s}$ <p>OR</p> <p>(b) $\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt} \Rightarrow \frac{3}{101} = \frac{-5}{5^2+x^2} \Big _{x=50} \times \frac{dx}{dt}$</p> $\Rightarrow \frac{3}{101} = \frac{-5}{2525} \times \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -15 \text{ m/s}$ <p>Hence the speed is 15 m/s</p>	$\frac{1}{2}$ $\frac{1}{2}$
37.	<p>According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.</p> <p>Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.</p>  <p>On the basis of the above information, answer the following questions :</p> <p>(i) Find the probability that an airplane reached its destination late. 2</p> <p>(ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence. 2</p>	
Sol.	<p>(i) Let A denote the event of airplane reaching its destination late</p> <p>$E_1 =$ severe turbulence</p> <p>$E_2 =$ moderate turbulence</p> <p>$E_3 =$ light turbulence</p> $P(A) = P(E_1)P(A E_1) + P(E_2)P(A E_2) + P(E_3)P(A E_3)$ $= \frac{1}{3} \times \frac{55}{100} + \frac{1}{3} \times \frac{37}{100} + \frac{1}{3} \times \frac{17}{100}$ $= \frac{1}{3} \left(\frac{109}{100} \right) = \frac{109}{300}$	$\left. \begin{array}{l} \frac{1}{2} \\ \frac{1}{2} \end{array} \right\}$ 1 $\frac{1}{2}$

(iii) (a)



OR

$$(b) f(x) = 2 \sin^{-1}(1-x)$$

$$-1 \leq 1-x \leq 1$$

$$\Rightarrow -2 \leq -x \leq 0$$

$$\Rightarrow 0 \leq x \leq 2$$

$$\text{Domain} = [0, 2]$$

$$\frac{-\pi}{2} \leq \sin^{-1}(1-x) \leq \frac{\pi}{2}$$

$$-\pi \leq 2 \sin^{-1}(1-x) \leq \pi$$

$$\text{So range} = [-\pi, \pi]$$

Correct
graph -
2

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$