Series : Z6YWX

SET~3

प्रश्न-पत्र कोड
Q.P. Code **65/6/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* — *अधिकतम अंक : **80***

*Time allowed : **3** hours* — *Maximum Marks : **80***

नोट	**NOTE**
(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।	(I) Please check that this question paper contains **23** printed pages.
(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।	(II) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।	(III) Please check that this question paper contains **38** questions.
(IV) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।**	(IV) **Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**
(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।	(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में **38** प्रश्न हैं। **सभी** प्रश्न **अनिवार्य** हैं।*

(ii) *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ**।*

(iii) ***खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **1** अंक के प्रश्न हैं।*

(iv) ***खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **2** अंकों के प्रश्न हैं।*

(v) ***खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **3** अंकों के प्रश्न हैं।*

(vi) ***खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **5** अंकों के प्रश्न हैं।*

(vii) ***खण्ड ङ** में प्रश्न संख्या **36** से **38** तक प्रकरण अध्ययन आधारित **4** अंकों के प्रश्न हैं।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

1. यदि $\tan^{-1}(x^2 - y^2) = a$, जहाँ 'a' एक अचर है, तो $\frac{dy}{dx}$ है :

(A) $\frac{x}{y}$ (B) $-\frac{x}{y}$

(C) $\frac{a}{x}$ (D) $\frac{a}{y}$

2. यदि $A = \begin{bmatrix} 0 & 0 & -5 \\ 0 & 3 & 0 \\ 4\cdot3 & 0 & 0 \end{bmatrix}$ है, तो A एक :

(A) विषम-सममित आव्यूह है (B) अदिश आव्यूह है

(C) विकर्ण आव्यूह है (D) वर्ग आव्यूह है

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\frac{dy}{dx}$ is :

(A) $\frac{x}{y}$ (B) $-\frac{x}{y}$

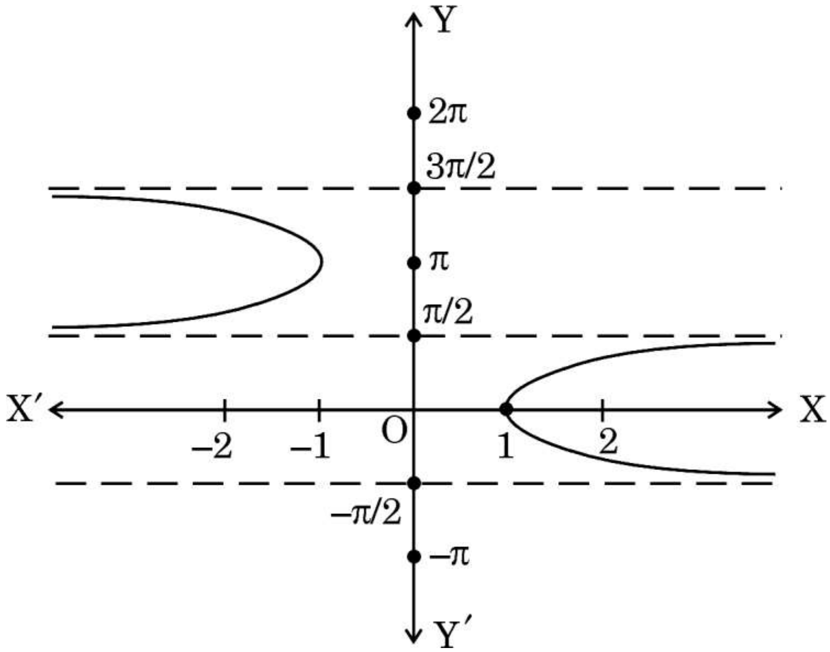
(C) $\frac{a}{x}$ (D) $\frac{a}{y}$

2. If $A = \begin{bmatrix} 0 & 0 & -5 \\ 0 & 3 & 0 \\ 4{\cdot}3 & 0 & 0 \end{bmatrix}$, then A is a :

(A) skew-symmetric matrix (B) scalar matrix

(C) diagonal matrix (D) square matrix

3. नीचे दिया गया ग्राफ़ दर्शाता है :



(A) $y = \sec^{-1} x$ (B) $y = \sec x$

(C) $y = \text{cosec}^{-1} x$ (D) $y = \text{cosec}\, x$

4. $\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ बराबर है :

(A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$

(C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

5. माना आव्यूह A और B के लिए AB' और $B'A$ दोनों परिभाषित हैं । यदि आव्यूह A की कोटि $n \times m$ है, तो आव्यूह B की कोटि है :

(A) $n \times n$ (B) $n \times m$

(C) $m \times m$ (D) $m \times n$

6. समान कोटि के दो विषम-सममित आव्यूहों का योगफल सदैव :

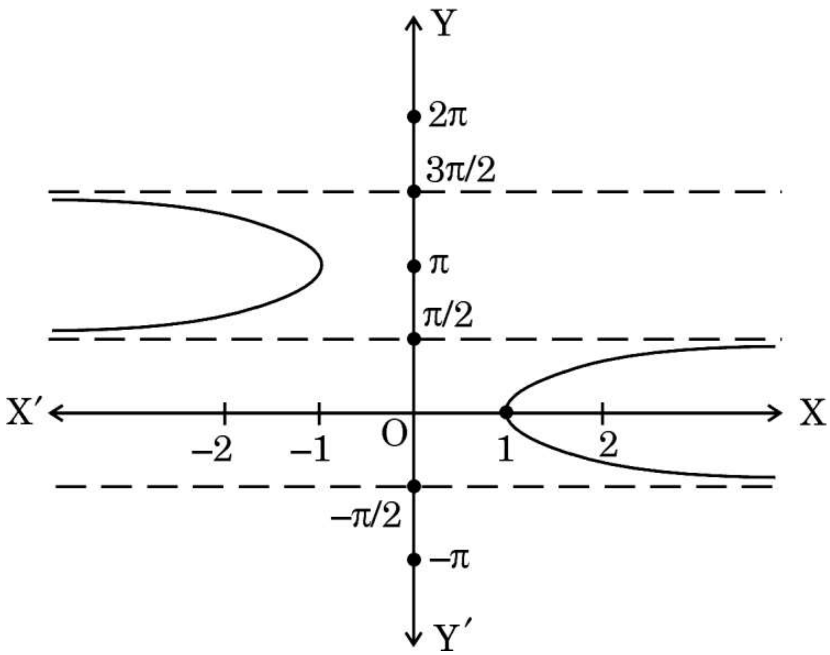
(A) एक विषम-सममित आव्यूह होता है

(B) एक सममित आव्यूह होता है

(C) एक शून्य आव्यूह होता है

(D) एक तत्समक आव्यूह होता है

3. The graph shown below depicts :



(A) $y = \sec^{-1} x$ (B) $y = \sec x$

(C) $y = \text{cosec}^{-1} x$ (D) $y = \text{cosec}\, x$

4. $\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to :

(A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$

(C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

5. Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is :

(A) $n \times n$ (B) $n \times m$

(C) $m \times m$ (D) $m \times n$

6. Sum of two skew-symmetric matrices of same order is always a/an :

(A) skew-symmetric matrix

(B) symmetric matrix

(C) null matrix

(D) identity matrix

7. यदि $y = a \cos(\log x) + b \sin(\log x)$ है, तो $x^2y_2 + xy_1$ है :

(A) $\cot(\log x)$ (B) y

(C) $-y$ (D) $\tan(\log x)$

8. यदि $f(x) = \begin{cases} \dfrac{\log(1+ax) + \log(1-bx)}{x}, & x \neq 0 \text{ के लिए} \\ k, & x = 0 \text{ के लिए} \end{cases}$

$x = 0$ पर संतत है, तो k का मान है :

(A) a (B) $a + b$

(C) $a - b$ (D) b

9. $f(x) = x^x$ का एक क्रांतिक बिंदु :

(A) $x = e$ पर है (B) $x = e^{-1}$ पर है

(C) $x = 0$ पर है (D) $x = 1$ पर है

10. अवकल समीकरण $\log\left(\dfrac{dy}{dx}\right) = 3x + 4y$ का हल है :

(A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$

(C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$

7. If $y = a \cos(\log x) + b \sin(\log x)$, then $x^2y_2 + xy_1$ is :

(A) $\cot(\log x)$ (B) y

(C) $-y$ (D) $\tan(\log x)$

8. If $f(x) = \begin{cases} \dfrac{\log(1+ax) + \log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$

is continuous at $x = 0$, then the value of k is :

(A) a (B) a + b

(C) a – b (D) b

9. $f(x) = x^x$ has a critical point at :

(A) $x = e$ (B) $x = e^{-1}$

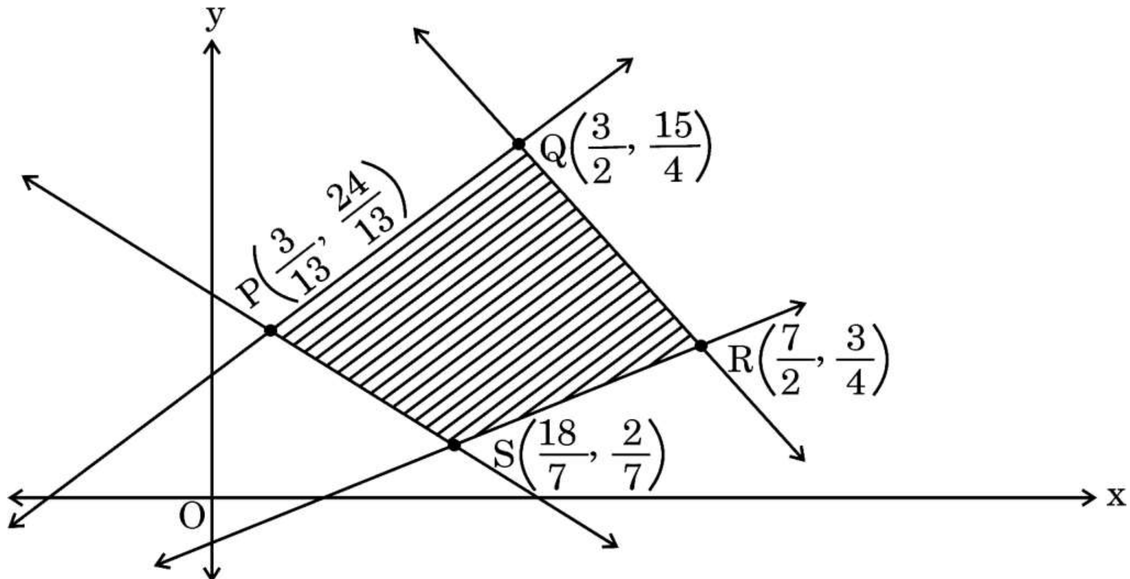
(C) $x = 0$ (D) $x = 1$

10. The solution for the differential equation $\log\left(\dfrac{dy}{dx}\right) = 3x + 4y$ is :

(A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$

(C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$

11. एक रैखिक प्रोग्रामन समस्या (LPP) के लिए, दिया गया उद्देश्य फलन $Z = x + 2y$ है। व्यवरोधों से बनने वाला सुसंगत क्षेत्र PQRS छायांकित क्षेत्र द्वारा आलेख में दिखाया गया है।



(ध्यान दें : आलेख पैमाने अनुसार नहीं है)

$$P \equiv \left(\frac{3}{13}, \frac{24}{13}\right), \quad Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right), \quad R \equiv \left(\frac{7}{2}, \frac{3}{4}\right), \quad S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$$

निम्नलिखित में से कौन-सा कथन सही है ?

(A) Z का न्यूनतम मान $S\left(\frac{18}{7}, \frac{2}{7}\right)$ पर है

(B) Z का अधिकतम मान $R\left(\frac{7}{2}, \frac{3}{4}\right)$ पर है

(C) (Z का मान P पर) > (Z का मान Q पर)

(D) (Z का मान Q पर) < (Z का मान R पर)

12. दिए गए अवकल समीकरण $\left[\frac{d^2y}{dx^2} - 1\right]^2 = \frac{dy}{dx}$ की कोटि और घात क्रमशः हैं :

(A) 2, 2 (B) 2, परिभाषित नहीं

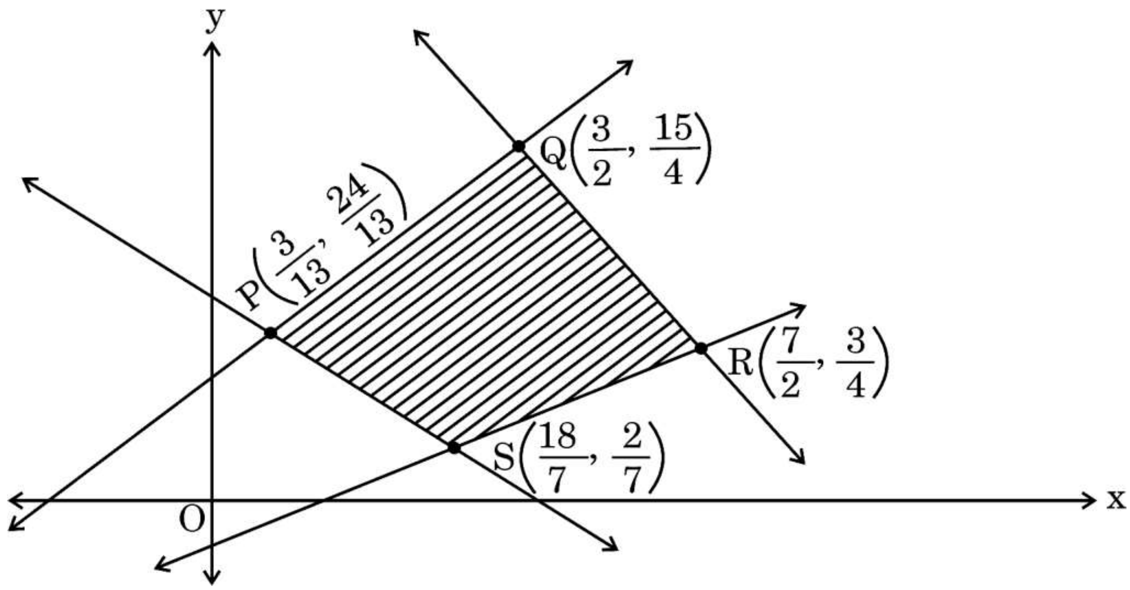
(C) 1, 2 (D) 1, परिभाषित नहीं

13. माना $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, $f(1) = 0$ है। तो, f(x) है :

(A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$ (B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$

(C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$

11. For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.



(Note : The figure is not to scale)

$$P \equiv \left(\frac{3}{13}, \frac{24}{13}\right), Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right), R \equiv \left(\frac{7}{2}, \frac{3}{4}\right), S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$$

Which of the following statements is correct ?

(A) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$

(B) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$

(C) (Value of Z at P) > (Value of Z at Q)

(D) (Value of Z at Q) < (Value of Z at R)

12. The order and degree of the differential equation $\left[\frac{d^2y}{dx^2} - 1\right]^2 = \frac{dy}{dx}$ are, respectively :

(A) 2, 2 (B) 2, not defined

(C) 1, 2 (D) 1, not defined

13. Let $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, $f(1) = 0$. Then, f(x) is :

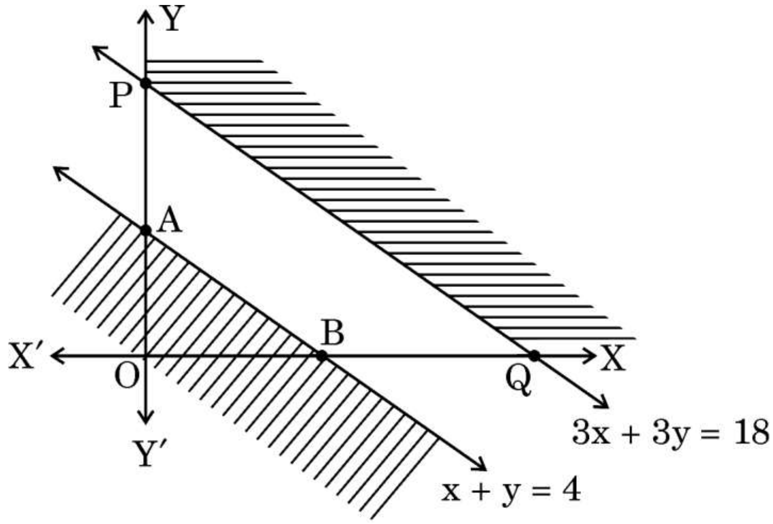
(A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$ (B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$

(C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$

14. एक रैखिक प्रोग्रामन समस्या (LPP) के लिए, उद्देश्य फलन $Z = 2x + 5y$ का निम्नलिखित व्यवरोधों के अंतर्गत अधिकतमीकरण करना है :

$$x + y \leq 4, \quad 3x + 3y \geq 18, \quad x, y \geq 0$$

आलेख का अध्ययन कीजिए और निम्नलिखित से सही विकल्प चुनिए।



(ध्यान दें : आलेख पैमाने अनुसार नहीं है)

दी गई रैखिक प्रोग्रामन समस्या का हल :

(A) छायांकित अपरिबद्ध क्षेत्र में है।

(B) त्रिभुज AOB में है।

(C) नहीं है।

(D) त्रिभुज AOB और छायांकित अपरिबद्ध क्षेत्र के इकट्ठे क्षेत्र में है।

15. वक्र $y^2 = x$, का $x = 0$ तथा $x = 1$ के बीच परिबद्ध क्षेत्र का क्षेत्रफल है :

(A) $\frac{3}{2}$ वर्ग इकाई

(B) $\frac{2}{3}$ वर्ग इकाई

(C) 3 वर्ग इकाई

(D) $\frac{4}{3}$ वर्ग इकाई

16. $\int \frac{x+5}{(x+6)^2} e^x \, dx$ बराबर है :

(A) $\log (x + 6) + C$

(B) $e^x + C$

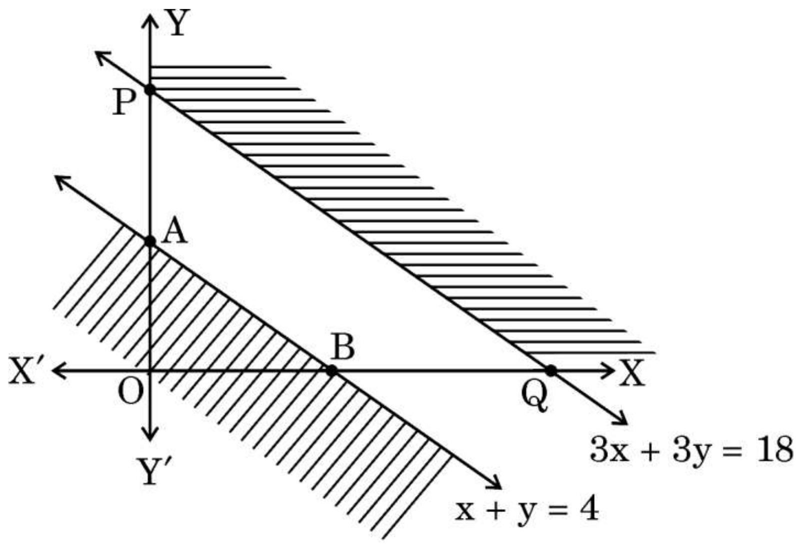
(C) $\frac{e^x}{x+6} + C$

(D) $\frac{-1}{(x+6)^2} + C$

14. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints :

$$x + y \leq 4, \quad 3x + 3y \geq 18, \quad x, y \geq 0$$

Study the graph and select the correct option.



(Note : The figure is not to scale)

The solution of the given LPP :

(A) lies in the shaded unbounded region.

(B) lies in Δ AOB.

(C) does not exist.

(D) lies in the combined region of Δ AOB and unbounded shaded region.

15. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is :

(A) $\frac{3}{2}$ sq units (B) $\frac{2}{3}$ sq units

(C) 3 sq units (D) $\frac{4}{3}$ sq units

16. $\int \frac{x+5}{(x+6)^2} e^x \, dx$ is equal to :

(A) $\log (x + 6) + C$ (B) $e^x + C$

(C) $\frac{e^x}{x+6} + C$ (D) $\frac{-1}{(x+6)^2} + C$

17. माना $|\vec{a}| = 5$ और $-2 \le \lambda \le 1$ है। तो, $|\lambda \vec{a}|$ का परिसर है :

(A) $[5, 10]$ (B) $[-2, 5]$

(C) $[-2, 1]$ (D) $[-10, 5]$

18. सभी तीन सदस्यों A, B और C के उपस्थित होने पर ही बैठक होगी। सदस्य A के उपस्थित नहीं होने की प्रायिकता 0·10, सदस्य B के उपस्थित नहीं होने की प्रायिकता 0·20 और सदस्य C के उपस्थित नहीं होने की प्रायिकता 0·05 है। बैठक रद्द होने की प्रायिकता है :

(A) 0·35 (B) 0·316

(C) 0·001 (D) 0·65

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं, जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए।*

(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है।

(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है।

19. *अभिकथन (A) :* यदि $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 256$ तथा $|\vec{b}| = 8$ है, तो $|\vec{a}| = 2$ है।

तर्क (R) : $\sin^2 \theta + \cos^2 \theta = 1$ तथा $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta$ और $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$ है।

20. *अभिकथन (A) :* माना $f(x) = e^x$ तथा $g(x) = \log x$ है। तो $(f + g)\, x = e^x + \log x$ है, जहाँ $(f + g)$ का प्रांत R है।

तर्क (R) : $(f + g)$ का प्रांत = (f) का प्रांत $\cap$ (g) का प्रांत।

17. Let $|\vec{a}| = 5$ and $-2 \le \lambda \le 1$. Then, the range of $|\lambda \vec{a}|$ is :

(A) [5, 10]
(B) [– 2, 5]
(C) [– 2, 1]
(D) [–10, 5]

18. A meeting will be held only if all three members A, B and C are present. The probability that member A does not turn up is 0·10, member B does not turn up is 0·20 and member C does not turn up is 0·05. The probability of the meeting being cancelled is :

(A) 0·35
(B) 0·316
(C) 0·001
(D) 0·65

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 256$ and $|\vec{b}| = 8$, then $|\vec{a}| = 2$.

Reason (R) : $\sin^2 \theta + \cos^2 \theta = 1$ and

$|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin \theta$ and $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos \theta$.

20. *Assertion (A) :* Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)\, x = e^x + \log x$ where domain of $(f + g)$ is R.

Reason (R) : $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.

खण्ड ख

इस खण्ड में 5 अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. (क) यदि बिंदुओं A और B के क्रमश: $\vec{a}$ और $\vec{b}$ स्थिति सदिश हों, तो बिंदु C, जो बढ़ाए गए BA पर स्थित है, का स्थिति सदिश ज्ञात कीजिए जबकि BC = 3BA हो।

अथवा

(ख) सदिश $\vec{r}$ तीन अक्षों x, y और z के साथ समान कोण पर झुका हुआ है। यदि $\vec{r}$ का परिमाण $5\sqrt{3}$ इकाई है, तो $\vec{r}$ ज्ञात कीजिए।

22. $f(x) = \sin^{-1}(-x^2)$ का प्रांत ज्ञात कीजिए।

23. अंतराल ज्ञात कीजिए जिसमें $f(x) = x + \frac{1}{x}, \quad x \neq 0$ में सदैव वर्धमान रहता है।

24. (क) $e^{\sqrt{2x}}$ के सापेक्ष $\sqrt{e^{\sqrt{2x}}}$ का अवकलज ज्ञात कीजिए, $x > 0$ के लिए।

अथवा

(ख) यदि $(x)^y = (y)^x$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

25. वह कोण ज्ञात कीजिए जिस पर दी गई रेखाएँ एक दूसरे पर झुकी हैं :

$$l_1 : \frac{x-5}{2} = \frac{y+3}{1} = \frac{z-1}{-3}$$

$$l_2 : \frac{x}{3} = \frac{y-1}{2} = \frac{z+5}{-1}$$

खण्ड ग

इस खण्ड में 6 लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. x का मान ज्ञात कीजिए, यदि $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O.$

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA.

OR

(b) Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.

22. Find the domain of $f(x) = \sin^{-1}(-x^2)$.

23. Find the interval in which $f(x) = x + \frac{1}{x}$ is always increasing, $x \neq 0$.

24. (a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$.

OR

(b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.

25. Find the angle at which the given lines are inclined to each other :

$$l_1 : \frac{x-5}{2} = \frac{y+3}{1} = \frac{z-1}{-3}$$

$$l_2 : \frac{x}{3} = \frac{y-1}{2} = \frac{z+5}{-1}$$

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. Find the value of x, if $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$.

27. (क) बिंदु P(2, 4, –1) की रेखा $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ से दूरी ज्ञात कीजिए।

अथवा

(ख) माना बिंदुओं A, B और C के स्थिति सदिश क्रमशः $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ और $\hat{i} + 5\hat{j} + 3\hat{k}$ हैं। बिंदु A से गुज़रने वाली और BC के समांतर रेखा के सदिश और कार्तीय समीकरण ज्ञात कीजिए।

28. रैखिक प्रोग्रामन समस्या पर विचार कीजिए, जहाँ उद्देश्य फलन $Z = (x + 4y)$ का निम्न व्यवरोधों

$$2x + y \geq 1000$$
$$x + 2y \geq 800$$
$$x, y \geq 0$$

के अंतर्गत न्यूनतमीकरण करना है।
सुसंगत क्षेत्र का एक स्वच्छ आलेख बनाइए और Z का न्यूनतम मान ज्ञात कीजिए।

29. (क) एक छात्र प्राकृत संख्याओं के युग्म (जोड़े) इस प्रकार बनाना चाहता है कि वे समीकरण $2x + y = 41$ को संतुष्ट करते हों, $x, y \in N$। इस तरह बने संबंध का प्रांत और परिसर ज्ञात कीजिए। जाँच कीजिए कि क्या यह संबंध स्वतुल्य, सममित अथवा संक्रामक है। अतः ज्ञात कीजिए कि क्या यह एक तुल्यता संबंध है अथवा नहीं।

अथवा

(ख) दर्शाइए कि $f(n) = \begin{cases} n-1, & \text{यदि } n \text{ सम है} \\ n+1, & \text{यदि } n \text{ विषम है} \end{cases}$

द्वारा प्रदत्त फलन $f : N \to N$, जहाँ N प्राकृत संख्याओं का समुच्चय है, एकैकी आच्छादी है।

30. (क) x के सापेक्ष $y = \sin^{-1}(3x - 4x^3)$ का अवकलज ज्ञात कीजिए, यदि $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ है।

अथवा

(ख) x के सापेक्ष $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ का अवकलज ज्ञात कीजिए, जब $x \in (0, 1)$ है।

27. (a) Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

OR

(b) Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.

28. Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints

$$2x + y \geq 1000$$

$$x + 2y \geq 800$$

$$x, y \geq 0.$$

Draw a neat graph of the feasible region and find the minimum value of Z.

29. (a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

OR

(b) Show that the function $f : N \to N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, & \text{if } n \text{ is even} \\ n+1, & \text{if } n \text{ is odd} \end{cases}$ is a bijection.

30. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

OR

(b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.

31. थैले I में 4 सफेद और 5 काली गेंदें हैं। थैले II में 6 सफेद और 7 काली गेंदें हैं। थैले I में से एक गेंद यादृच्छया निकाल कर थैले II में डाल दी जाती है और फिर एक गेंद थैले II में से यादृच्छया निकाली जाती है। इस गेंद के सफेद रंग की होने की प्रायिकता ज्ञात कीजिए।

खण्ड घ

इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. (क) अवकल समीकरण $x^2y\,dx - (x^3 + y^3)\,dy = 0$ का हल ज्ञात कीजिए।

अथवा

(ख) अवकल समीकरण $(1 + x^2)\,\dfrac{dy}{dx} + 2xy - 4x^2 = 0$ की प्रारंभिक स्थिति $y(0) = 0$ के अंतर्गत हल ज्ञात कीजिए।

33. समाकलन के प्रयोग से दीर्घवृत्त $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ तथा रेखाओं $x = -\dfrac{a}{2}, x = \dfrac{a}{2}$ के बीच परिबद्ध क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

34. (क) ज्ञात कीजिए :

$$\int \frac{x^2 + 1}{(x-1)^2\,(x+3)}\,dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\,dx$$

35. दर्शाइए कि बिन्दुओं A (0, –1, –1) और B (4, 5, 1) से गुज़रने वाली रेखा, बिन्दुओं C (3, 9, 4) और D (– 4, 4, 4) से गुज़रने वाली रेखा को काटती है।

31. Bag I contains 4 white and 5 black balls. Bag II contains 6 white and 7 black balls. A ball drawn randomly by from bag I is transferred to bag II and then a ball is drawn randomly from bag II. Find the probability that the ball drawn is white.

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Solve the differential equation : $x^2y\,dx - (x^3 + y^3)\,dy = 0$.

OR

(b) Solve the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.

33. Using integration, find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ bounded between the lines $x = -\frac{a}{2}$ to $x = \frac{a}{2}$.

34. (a) Find :

$$\int \frac{x^2 + 1}{(x - 1)^2\,(x + 3)}\,dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\,dx$$

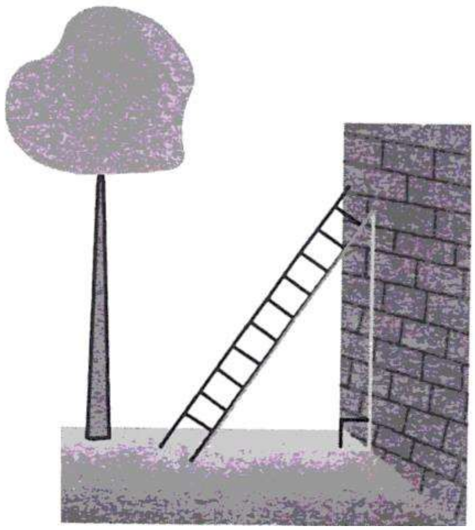
35. Show that the line passing through the points A (0, –1, –1) and B (4, 5, 1) intersects the line joining points C (3, 9, 4) and D (– 4, 4, 4).

खण्ड ङ

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36.



निश्चित लंबाई 'h' की एक सीढ़ी दीवार के साथ इस प्रकार लगाई जाती है कि वह दीवार की ऊँचाई के साथ चलने के लिए स्वतंत्र है।
उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) एक निश्चित क्षण में दीवार और सीढ़ी के निचले हिस्से के बीच की दूरी (y) को 'h' और दीवार पर ऊँचाई (x) के रूप में व्यक्त कीजिए। इसके अलावा, एक पर्यवेक्षक द्वारा किनारे से देखे गए समकोण त्रिभुज के क्षेत्रफल (A) के लिए एक अभिव्यक्ति, h तथा x के पदों में लिखिए। *1*

(ii) दीवार पर ऊँचाई (x) के सापेक्ष, क्षेत्रफल (A) का अवकलज ज्ञात कीजिए और इसका क्रांतिक बिंदु भी ज्ञात कीजिए। *1*

(iii) (क) दर्शाइए कि क्रांतिक बिंदु पर, समकोण त्रिभुज का क्षेत्रफल (A) अधिकतम होगा। *2*

अथवा

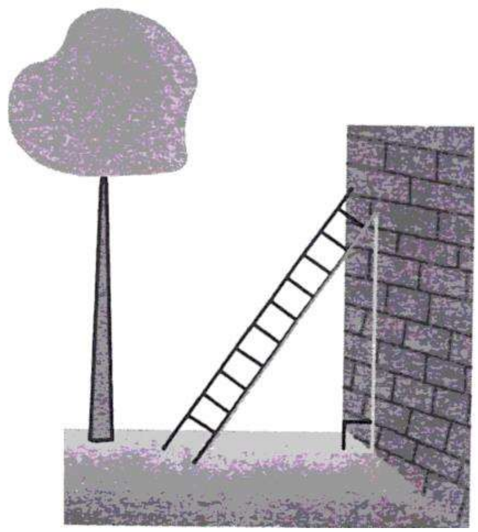
(iii) (ख) यदि 5 m लम्बी सीढ़ी का पाद, दीवार की ओर इस प्रकार खींचा जा रहा है कि दूरी (y) घटने की दर 2 m/s है, तो दीवार पर ऊँचाई (x) किस दर से बढ़ रही है, जब सीढ़ी का पाद दीवार से 3 m दूर है ? *2*

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36.



A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions :

(i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. *1*

(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point. *1*

(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. *2*

OR

(iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ? *2*

प्रकरण अध्ययन – 2

37. इलेक्ट्रॉनिक सामान बेचने वाली एक दुकान, केवल तीन प्रतिष्ठित कंपनियों A, B और C के स्मार्टफोन बेचती है क्योंकि उनके दोषपूर्ण स्मार्टफोन बनाने की प्रायिकताएँ केवल क्रमश: 5%, 4% और 2% हैं। उसकी सूची में, उसके पास कंपनी A के 25% स्मार्टफोन, कंपनी B के 35% स्मार्टफोन और कंपनी C के 40% स्मार्टफोन हैं।

एक व्यक्ति इस दुकान से एक स्मार्टफोन खरीदता है।

(i) इस फोन के दोषपूर्ण होने की प्रायिकता ज्ञात कीजिए। *2*

(ii) यह दोषपूर्ण स्मार्टफोन, कंपनी B के द्वारा उत्पादित किए जाने की प्रायिकता क्या है ? *2*

प्रकरण अध्ययन – 3

38. तीन विद्यार्थी नेहा, रानी और सैम स्टेशनरी का सामान खरीदने के लिए बाज़ार जाते हैं। नेहा ने 4 पेन, 3 कापियाँ और 2 रबड़, ₹ 60 देकर खरीदे। रानी ने 2 पेन, 4 कापियाँ और 6 रबड़, ₹ 90 देकर खरीदे। सैम ने ₹ 70 में 6 पेन, 2 कापियाँ और 3 रबड़ खरीदे।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) प्रत्येक वस्तु का मूल्य ज्ञात करने की समस्या को हल करने के लिए आवश्यक समीकरण बनाइए और इनको आव्यूह रूप AX = B में व्यक्त कीजिए। *1*

(ii) $|A|$ ज्ञात कीजिए और पुष्टि कीजिए कि क्या A^{-1} ज्ञात करना संभव है। *1*

(iii) (क) यदि संभव है, तो A^{-1} ज्ञात कीजिए और X ज्ञात करने के लिए सूत्र लिखिए। *2*

अथवा

(iii) (ख) $A^2 - 8I$, जहाँ I एक तत्समक आव्यूह है, ज्ञात कीजिए। *2*

Case Study – 2

37. A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

A person buys a smartphone from this shop.

(i) Find the probability that it was defective. *2*

(ii) What is the probability that this defective smartphone was manufactured by company B ? *2*

Case Study – 3

38. Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.

Based upon the above information, answer the following questions :

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B. *1*

(ii) Find $|A|$ and confirm if it is possible to find A^{-1}. *1*

(iii) (a) Find A^{-1}, if possible, and write the formula to find X. *2*

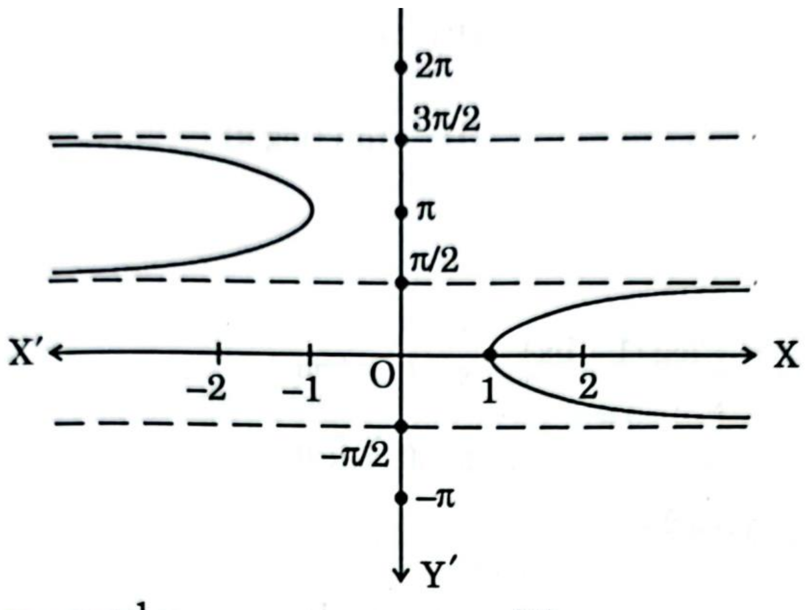
OR

(iii) (b) Find $A^2 - 8I$, where I is an identity matrix. *2*

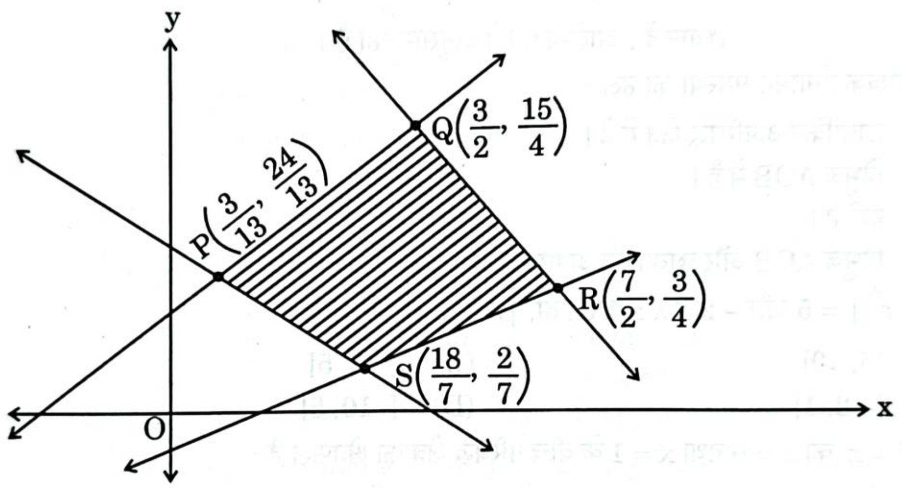
Marking Scheme **Strictly Confidential** **(For Internal and Restricted use only)** **Senior Secondary Examination, 2025** **SUBJECT NAME MATHEMATICS (Q.P. CODE – 65/6/3)**	
General Instructions: -	
1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.

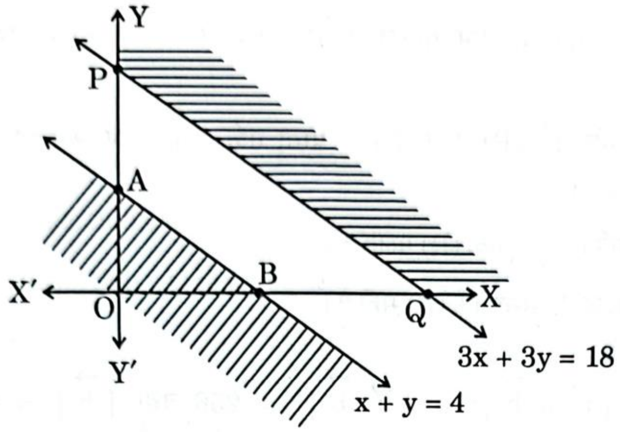
10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks_____________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of the answer marked correct and the rest as wrong, but no marks
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME – 65/6/3

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A This section comprises multiple choice questions (MCQs) of 1 mark each.	
1.	If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\frac{dy}{dx}$ is : (A) $\frac{x}{y}$ (B) $-\frac{x}{y}$ (C) $\frac{a}{x}$ (D) $\frac{a}{y}$	
Ans	(A) $\frac{x}{y}$	1
2.	If $A = \begin{bmatrix} 0 & 0 & -5 \\ 0 & 3 & 0 \\ 4\cdot3 & 0 & 0 \end{bmatrix}$, then A is a : (A) skew-symmetric matrix (B) scalar matrix (C) diagonal matrix (D) square matrix	
Ans	(D) square matrix	1
3.	 (A) $y = \sec^{-1} x$ (B) $y = \sec x$ (C) $y = \text{cosec}^{-1} x$ (D) $y = \text{cosec } x$	
Ans	(A) $y = \sec^{-1}x$	1

4.	$\left[\sec^{-1}(-\sqrt{2})-\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to : (A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$ (C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$	
Ans	(D) $\frac{7\pi}{12}$	1
5.	Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is : (A) $n \times n$ (B) $n \times m$ (C) $m \times m$ (D) $m \times n$	
Ans	(B) $n \times m$	1
6.	Sum of two skew-symmetric matrices of same order is always a/an : (A) skew-symmetric matrix (B) symmetric matrix (C) null matrix (D) identity matrix	
Ans	(A) skew-symmetric matrix	1
7.	If $y = a\cos(\log x) + b\sin(\log x)$, then $x^2y_2 + xy_1$ is : (A) $\cot(\log x)$ (B) y (C) $-y$ (D) $\tan(\log x)$	
Ans	(C) $-y$	1
8.	If $f(x) = \begin{cases} \frac{\log(1+ax)+\log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is : (A) a (B) a + b (C) a – b (D) b	
Ans	(C) a – b	1
9.	$f(x) = x^x$ has a critical point at : (A) $x = e$ (B) $x = e^{-1}$ (C) $x = 0$ (D) $x = 1$	
Ans	(B) $x = e^{-1}$	1
10.	The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is : (A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$ (C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$	
Ans	(D) $3e^{-4y} + 4e^{3x} + 12C = 0$	1

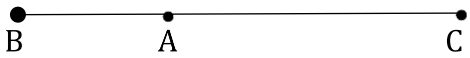
11.	For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.  (Note : The figure is not to scale) $P \equiv \left(\frac{3}{13}, \frac{24}{13}\right), Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right), R \equiv \left(\frac{7}{2}, \frac{3}{4}\right), S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$ Which of the following statements is correct ? (A) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$ (B) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$ (C) (Value of Z at P) > (Value of Z at Q) (D) (Value of Z at Q) < (Value of Z at R)	
Ans	(A) Z is minimum at S $\left(\frac{18}{7}, \frac{2}{7}\right)$	1
12.	The order and degree of the differential equation $\left[\frac{d^2y}{dx^2} - 1\right]^2 = \frac{dy}{dx}$ are, respectively : (A) 2, 2 (B) 2, not defined (C) 1, 2 (D) 1, not defined	
Ans	(A) 2, 2	1
13.	Let $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, $f(1) = 0$. Then, f(x) is : (A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$ (B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$ (C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$	
Ans	(B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$	1

14.	In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints : $x + y \leq 4,\ 3x + 3y \geq 18,\ x, y \geq 0$ Study the graph and select the correct option.  (Note : The figure is not to scale) The solution of the given LPP : (A) lies in the shaded unbounded region. (B) lies in Δ AOB. (C) does not exist. (D) lies in the combined region of Δ AOB and unbounded shaded region.	
Ans	(C) does not exist	1
15.	The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is : (A) $\frac{3}{2}$ sq units (B) $\frac{2}{3}$ sq units (C) 3 sq units (D) $\frac{4}{3}$ sq units	
Ans	(D) $\frac{4}{3}$ sq units	1
16.	$\int \frac{x+5}{(x+6)^2} e^x \, dx$ is equal to : (A) $\log(x+6) + C$ (B) $e^x + C$ (C) $\frac{e^x}{x+6} + C$ (D) $\frac{-1}{(x+6)^2} + C$	
Ans	(C) $\frac{e^x}{x+6} + C$	1
17.	Let $\lvert \vec{a} \rvert = 5$ and $-2 \leq \lambda \leq 1$. Then, the range of $\lvert \lambda \vec{a} \rvert$ is : (A) [5, 10] (B) [– 2, 5] (C) [– 2, 1] (D) [–10, 5]	
Ans	1 mark for any attempt as correct answer is not given in any option	1

18.	A meeting will be held only if all three members A, B and C are present. The probability that member A does not turn up is 0·10, member B does not turn up is 0·20 and member C does not turn up is 0·05. The probability of the meeting being cancelled is : (A) 0·35 (B) 0·316 (C) 0·001 (D) 0·65	
Ans	(B) 0.316	1
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	**19.** *Assertion (A) :* If $\vert \vec{a} \times \vec{b} \vert^2 + \vert \vec{a} \cdot \vec{b} \vert^2 = 256$ and $\vert \vec{b} \vert = 8$, then $\vert \vec{a} \vert = 2$. *Reason (R) :* $\sin^2 \theta + \cos^2 \theta = 1$ and $\vert \vec{a} \times \vec{b} \vert = \vert \vec{a} \vert \vert \vec{b} \vert \sin \theta$ and $\vec{a} \cdot \vec{b} = \vert \vec{a} \vert \vert \vec{b} \vert \cos \theta$.	
Ans	(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).	1
20.	**20.** *Assertion (A) :* Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)\, x = e^x + \log x$ where domain of $(f + g)$ is R. *Reason (R) :* $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.	
Ans	(D) Assertion (A) is false, but Reason (R) is true.	1

SECTION-B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21.	**(a)** If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA. **OR** **(b)** Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.	
Ans	(a) C divides BA in the ratio 3 :2 externally  Required vector = $\vec{c} = \frac{3\vec{a}-2\vec{b}}{3-2} = 3\vec{a}-2\vec{b}$ OR (b) Unit vector equally inclined to coordinate axes is $\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$ $\vec{r} = 5\sqrt{3}(\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}) = 5\hat{i} + 5\hat{j} + 5\hat{k}$ (or $-5\hat{i} - 5\hat{j} - 5\hat{k}$)	1 1 1 1
22.	**Find the domain of $f(x) = \sin^{-1}(-x^2)$.**	
Ans	(a) $-1 \le -x^2 \le 1 \Rightarrow -1 \le -x^2 \le 0$ $\Rightarrow 0 \le x^2 \le 1 \Rightarrow -1 \le x \le 1$	1 1
23.	**Find the interval in which $f(x) = x + \frac{1}{x}$ is always increasing, $x \ne 0$.**	
Ans	$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$ f '(x) = 0 gives x = 1, -1 f is decreasing in (-1, 0) U (0, 1) as f '(x) < 0 f is increasing in R - (-1, 1) as f '(x) > 0	1 1 1
24.	**(a)** Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for x > 0. **OR** **(b)** If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.	
Ans	(a) u= $\sqrt{e^{\sqrt{2x}}}$ and v = $e^{\sqrt{2x}}$ Derivative of $\sqrt{v}$ wrt v = $\frac{1}{2\sqrt{v}}$. Required derivative = $\frac{1}{2\sqrt{e^{\sqrt{2x}}}}$. OR	½ 1 ½

<table>
<tr><td></td><td>Taking log on both sides, we get y logx = x logy

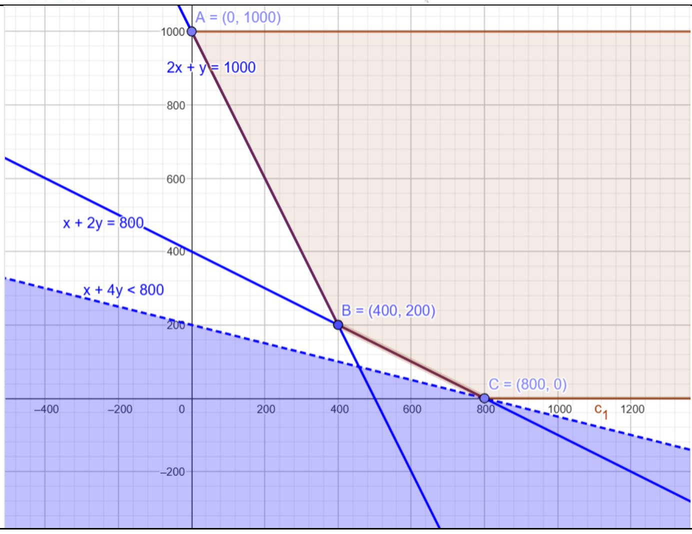
Differentiating both sides w.r.t. x, we get
$$\frac{y}{x.} + \log x\frac{dy}{dx} = \frac{x}{y.}\frac{dy}{dx} + \log y$$
$$\Rightarrow \frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$$</td><td>½

1

½</td></tr>
<tr><td>25.</td><td>Find the angle at which the given lines are inclined to each other :
$$l_1 : \frac{x-5}{2} = \frac{y+3}{1} = \frac{z-1}{-3}$$
$$l_2 : \frac{x}{3} = \frac{y-1}{2} = \frac{z+5}{-1}$$</td><td></td></tr>
<tr><td>Ans</td><td>Let θ be the angle between given lines
$$\therefore \cos\theta = \frac{(2\times 3) + (1\times 2) + (-3\times -1)}{\sqrt{14}\sqrt{14}}$$
$$\Rightarrow \cos\theta = \frac{11}{14}$$
$$\Rightarrow \theta = \cos^{-1}\left(\frac{11}{14}\right)$$</td><td>1
½
½</td></tr>
<tr><td colspan="3">SECTION-C
This section comprises 6 Short Answer (SA) type questions of 3 marks each.</td></tr>
<tr><td>26.</td><td>Find the value of x, if $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O.$</td><td></td></tr>
<tr><td>Ans</td><td>$$\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$$
$$\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 7+2x \\ 12+x \\ 21+2x \end{bmatrix} = 0$$
$$x^2 + 16x + 28 = 0$$
$$\Rightarrow (x+14)(x+2) = 0$$
$$\Rightarrow x = -14, x = -2$$</td><td>

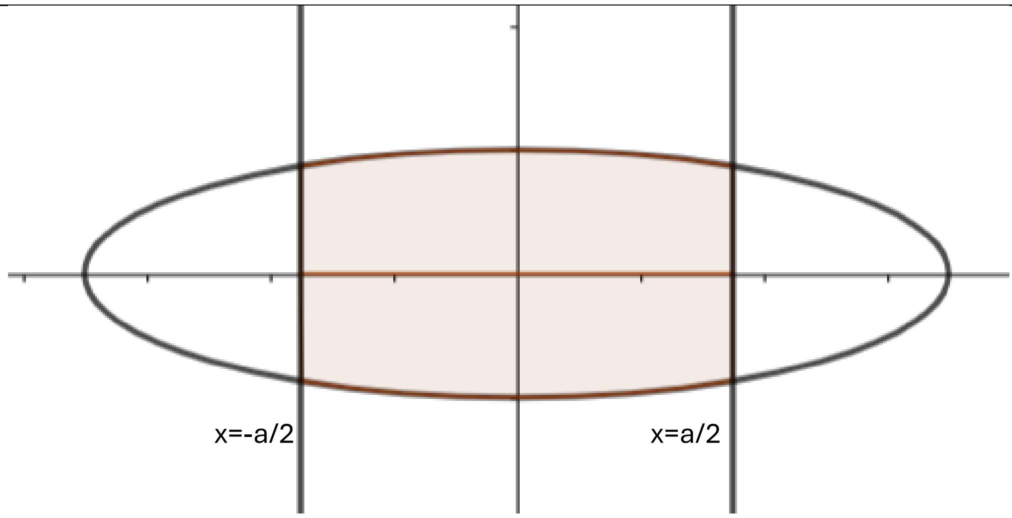
1
1

1</td></tr>
</table>

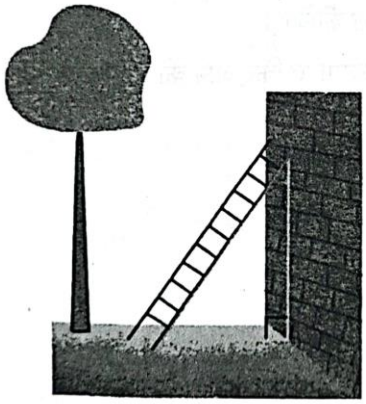
27.	(a) Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. **OR** (b) Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.	
Ans	(a) Let $\vec{a}_2 = 2\hat{i} + 4\hat{j} - \hat{k}$, $\vec{a}_1 = -5\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$ Distance between point and line is given by $d = \frac{\lvert(\vec{a_2} - \vec{a}_1)\times\vec{b}\rvert}{\lvert\vec{b}\rvert}$ Here $(\vec{a}_2 - \vec{a}_1) = 7\hat{i} + 7\hat{j} - 7\hat{k}$ $(\vec{a}_2 - \vec{a}_1)\times\vec{b} = -35\hat{i} + 56\hat{j} + 21\hat{k}$ $d = \frac{49\sqrt{2}}{7\sqrt{2}} = 7$ OR (b) Direction vector of line $= 3\hat{j} + 4\hat{k}$ Vector equation is $\vec{r} = 3\hat{i} - \hat{j} - 2\hat{k} + \mu(3\hat{j} + 4\hat{k})$ Cartesian equation is $\frac{x-3}{0} = \frac{y+1}{3} = \frac{z+2}{4}$	 ½ 1½ 1 1 1 1
28.	Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints $2x + y \geq 1000$ $x + 2y \geq 800$ $x, y \geq 0$. Draw a neat graph of the feasible region and find the minimum value of Z.	
Ans		

	Correct Graph and shading Corner points Value of Z (800, 0) 800 minimum (400, 200) 1200 (0, 1000) 4000 $x + 4y < 800$ has no region common with feasible region, hence 800 is minimum	1½ 1 ½
29.	(a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not. **OR** (b) Show that the function $f : N \to N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, \text{ if n is even} \\ n+1, \text{ if n is odd} \end{cases}$ is a bijection.	
Ans	(a) R ={(1,39), (2, 37), _ _ _, (20, 1)} Domain = {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20} Range = {1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39} (1, 1) does not belong to R hence not reflexive (1, 39) belongs to R but (39, 1) does not belong to R hence not symmetric (11, 19) and (19, 3) belong to R but (11, 3) does not belong to R hence not transitive Hence, R is not an equivalence relation. OR (a) Let f(x) = f(y) Let x and y are both odd or both even Then either x+1 = y + 1 or x-1 = y-1 gives x = y x odd and y even is rejected as x + 1 = y -1 gives x – y = -2 not possible as odd number and even number cannot differ by 2 Hence f is one-one For onto: Let f(x) = y gives x = y + 1 or x = y - 1 If y is odd, x is even and if y is even, x is odd Range = N = co-domain, hence onto As f is both one-one and onto hence bijective	½ ½ ½ ½ 1 ½ ½ ½ 1 ½

<table>
<tr><td>30.</td><td>(a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.
OR
(b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.</td><td></td></tr>
<tr><td>Ans</td><td>(a) x = sint gives $y = \sin^{-1}(\sin 3t) = 3t = 3\sin^{-1}x$
$\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$
Aliter: $\frac{dy}{dx} = \frac{3-12x^2}{\sqrt{1-(3x-4x^3)^2}}$
OR
(b) x = tant gives $y = \cos^{-1}(\cos 2t) = 2t = 2\tan^{-1}x$
$\frac{dy}{dx} = \frac{2}{1+x^2}$
Aliter: $\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-4x}{(1+x^2)^2}$.</td><td>$\frac{1}{2}+1+\frac{1}{2}$
1
3
$\frac{1}{2}+1+\frac{1}{2}$
1
3</td></tr>
<tr><td>31.</td><td>Bag I contains 4 white and 5 black balls. Bag II contains 6 white and 7 black balls. A ball drawn randomly by from bag I is transferred to bag II and then a ball is drawn randomly from bag II. Find the probability that the ball drawn is white.</td><td></td></tr>
<tr><td>Ans</td><td>P(white ball transferred) = $\frac{4}{9}$, Probability(black ball transferred) = $\frac{5}{9}$
P(white ball drawn from bag II) = $\frac{4}{9} \cdot \frac{7}{14} + \frac{5}{9} \cdot \frac{6}{14}$
$= \frac{29}{63}$</td><td>1
$1\frac{1}{2}$
½</td></tr>
<tr><td></td><td>SECTION-D
This section comprises 4 Long Answer (LA) type questions of 5 marks each.</td><td></td></tr>
<tr><td>32.</td><td>(a) Solve the differential equation : $x^2y\,dx - (x^3 + y^3)\,dy = 0$.
OR
(b) Solve the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition y(0) = 0.</td><td></td></tr>
<tr><td>Ans</td><td>(a) Given differential equation can be written as
$\frac{dy}{dx} = \frac{yx^2}{x^3+y^3}$--------- (i)
Let $y = vx \Rightarrow \frac{dv}{dx} = v + x\frac{dv}{dx}$ substituting in (i) we get</td><td>½
1</td></tr>
</table>

	$v + x\frac{dv}{dx} = \frac{vx^3}{x^3+v^3x^3} = \frac{v}{1+v^3}$ $x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$ $\left(\frac{1}{v^4}+\frac{1}{v}\right) dv = \frac{-dx}{x}$ Integrating we get $\frac{-1}{3v^3} + \log\lvert v\rvert = -\log\lvert x\rvert + C$ $\frac{-x^3}{3y^3} + \log\lvert y\rvert = C$ OR (b) Given D.E. is $\frac{dy}{dx} + \frac{2x}{1+x^2}\,y = \frac{4x^2}{1+x^2}$ Integrating factor is $e^{\int\frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$ Solution is $y(1+x^2) = \int 4x^2dx + C$ $y(1+x^2) = \frac{4x^3}{3} + C$ $y(0) = 0$ gives $C = 0$, hence solution is $y(1+x^2) = \frac{4x^3}{3}$	1 1 1 ½ 1 1 1 1 1
33.	**Using integration, find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ bounded between the lines $x = -\frac{a}{2}$ to $x = \frac{a}{2}$.**	
Ans	 Correct Graph: Required area = $\frac{4b}{a}\int_0^{\frac{a}{2}}\sqrt{a^2 - x^2\,dx}$	1 2

	$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{\frac{a}{2}}$ $= \frac{4b}{a}\left[\frac{a}{4}\cdot\frac{\sqrt{3}a}{2}+\frac{a^2}{2}\cdot\frac{\pi}{6}\right]$ $= ab\left[\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right]$	1 1
34.	(a) **Find :** $\int \frac{x^2+1}{(x-1)^2(x+3)}\,dx$ **OR** (b) **Evaluate :** $\int_0^{\pi/2} \frac{x}{\sin x+\cos x}\,dx$	
Ans	(a) $\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+3} = \frac{3/8}{x-1}+\frac{1/2}{(x-1)^2}+\frac{5/8}{x+3}$ $I = \frac{3}{8}\log\lvert x-1\rvert - \frac{1}{2(x-1)}+\frac{5}{8}\log\lvert x+3\rvert + C$ OR (b) Let $I = \int_0^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x}\,dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}-x}{\sin x+\cos x}\,dx$ $2I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}}{\sin x+\cos x}\,dx$ $= \frac{\pi}{2}\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}} \frac{1}{\sin\left(\frac{\pi}{4}+x\right)}\,dx$ $2I = \frac{\pi}{2\sqrt{2}}\log\left\lvert \operatorname{cosec}\left(\frac{\pi}{4}+x\right)-\cot\left(\frac{\pi}{4}+x\right)\right\rvert_0^{\frac{\pi}{2}}$ $I = \frac{\pi}{4\sqrt{2}}\log\frac{\sqrt{2}+1}{\sqrt{2}-1}$	1 + 2 2 1 1 1 1 1
35.	**Show that the line passing through the points A (0, –1, –1) and B (4, 5, 1) intersects the line joining points C (3, 9, 4) and D (– 4, 4, 4).**	
Ans	Drs of line passing through points A and B are $< 4,6,2 >$	1

	Drs of line passing through C and D are <-7, -5, 0> Consider $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3-0 & 9-(-1) & 4-(-1) \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}$ $= 3(+10) - 10(+14) + 5(22)$ $= 0$ Hence lines intersect each other	1 2 1
	SECTION-E This section comprises 3 case study-based questions of 4 marks each	
36.	**Case Study – 2**  A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall. Based upon the above information, answer the following questions : (i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. *1* (ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point. *1* (iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. *2* **OR** (iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ? *2*	
Ans	(i) $y^2 = h^2 - x^2$	½

	$A = \frac{1}{2}xy = \frac{1}{2}x\sqrt{h^2 - x^2}$ ii) $\frac{dA}{dx} = \frac{1}{2}\sqrt{h^2 - x^2} + \frac{1}{2}x\frac{-x}{\sqrt{h^2-x^2}}$ $\frac{dA}{dx} = 0$ gives $x = \frac{h}{\sqrt{2}}$ (iii)(a) $A'' = \frac{1}{2}\frac{-4x.\sqrt{h^2-x^2}-(h^2-2x^2)\frac{-x}{\sqrt{h^2-x^2}}}{h^2-x^2}$ is < 0 at $x = \frac{h}{\sqrt{2}}$ Hence A is maximum at critical point OR (iii)(b) $y^2 = 25 - x^2$ hence y = 3 gives x = 4 $2y\frac{dy}{dt} = -2x\frac{dx}{dt}$ $\frac{dx}{dt} = 1.5 m/s$	½ ½ ½ $1\frac{1}{2}$ ½ ½ 1 ½
37.	**A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.** **A person buys a smartphone from this shop.** **(i) Find the probability that it was defective.** ***2*** **(ii) What is the probability that this defective smartphone was manufactured by company B ?** ***2***	
Ans	(i) P(Defective) = $0.25\times 0.05 + 0.35\times 0.04 + 0.40\times 0.02$ $= 0.0345$ (ii) P(B/ Defective) = $\frac{0.35\times0.04}{0.25\times 0.05 + 0.35\times 0.04 + 0.40.\times 0.02}$ $= \frac{140}{345}$ or $\frac{28}{69}$	$1\frac{1}{2}$ ½ $1\frac{1}{2}$ ½

38.	Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers. Based upon the above information, answer the following questions : (i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B. *1* (ii) Find $\lvert A \rvert$ and confirm if it is possible to find A^{-1}. *1* (iii) (a) Find A^{-1}, if possible, and write the formula to find X. *2* **OR** (iii) (b) Find $A^2 - 8I$, where I is an identity matrix. *2*	
Ans	(i)Let the price of each pen, notepad, eraser be ₹x, ₹y and ₹z respectively Given system in the form AX = B is $\begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 60 \\ 90 \\ 70 \end{pmatrix}$ (ii)$\lvert A \rvert = 50 \neq 0$, hence A^{-1}exists (iii) (a) $A^{-1}=\frac{\text{adj}A}{\lvert A \rvert} = \frac{1}{50}\begin{pmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{pmatrix}$ $X = A^{-1}B$ **OR** (iii)(b) $A^2 = \begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 34 & 28 & 32 \\ 52 & 34 & 46 \\ 46 & 32 & 33 \end{pmatrix}$ $A^2 - 8I = \begin{pmatrix} 26 & 28 & 32 \\ 52 & 26 & 46 \\ 46 & 32 & 25 \end{pmatrix}$	1 1 1 ½ ½ 1 ½ ½