Chapter-13

Probability

The salient features of the chapter are –

- The conditional probability of an event E, given the occurrence of the event F is given by

$$P(E\,|F) = \frac{P(E \cap F)}{P(F)},\ P(F) \neq 0$$

$$0 \le P\,(E\,|\,F) \le 1,$$
$$P\,(E'\,|\,F\,) = 1 - P\,(E\,|\,F)$$
$$P\,((E \cup F)|G) = P\,(E|G) + P\,(F\,|\,G) - P\,((E \cap F)\,|\,G)$$

- $P\,(E \cap \quad) = \quad (E) \quad (\quad|\quad), \quad (E) \neq 0$
  $P\,(E \cap F) = P\,(F)P\,(E\,|\,F),\ P\,(F) \neq 0$

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  $P\,(F\,|\,E) = P\,(F),\ P(E) \neq 0$

- **Theorem of total probability:**

  Let $\{E_1,\ E_2,\ \ldots, E_n)$ be a partition of a sample space and suppose that each of $E_1,\ E_2,\ \ldots, E_n$ has non zero probability. Let A be any event associated with S, then

  $$P(A) = P(E_1)P(A\,|\,E_1) + P(E_2) + P(A\,|\,E_2) + .\ \ldots\ldots + P(E_n)P(A\,|\,E_n)$$

- **Bayes' theorem:** If $E_1,\ E_2,\ \ldots, E_n$ are events which constitute a partition of sample space S, i.e. $E_1,\ E_2,\ \ldots, E_n$ are pairwise disjoint and $E_1 4,\ E_2 4,\ \ldots, 4E_n = S$ *and* A be any event with non-zero probability, then, $P(E_i\,|A) = \dfrac{P(E_i \qquad\qquad i}{\sum_{j\ 1}^{n} P(E_i)\ P(A\,|\,E_i)}$

- A random variable is a real valued function whose domain is the sample space of a random experiment.

- The probability distribution of a random variable X is the system of numbers

1 2 n

1 2 n

Where, $p_i > 0, \sum_{i=1}^{n} p_i = 1,\ i = 1, 2, \ldots, n$

- Let X be a random variable whose possible values $x_1, x_2, x_3 \ldots \ldots, x_n$ occur with probabilities $p_1, p_2, p_3 \ldots \ldots, p_n$ respectively. The mean of X, denoted by μ is the number $\sum_{i=1}^{n} x_i p_i$. The mean of a random variable X is also called the expectation of X, denoted by E (X).
- Let X be a random variable whose possible values $x_1, x_2, x_3 \ldots \ldots, x_n$ occur with probabilities $p(x_1), p(x_2), \ldots p(x_n)$ respectively. Let $\mu = E(X)$ be the mean of X. The variance of X, denoted by Var (X) or σ_x is defined as $x^2\ Var\ (X) = \sum_{i=1}^{n} (x_i\ \mu)^2\ \ p(x_i)$ or equivalently $\sigma_x^2 = E\ (X - \mu)^2$. The non-negative number, $_x\sqrt{Var\ (X)} = \sqrt{\sum_{1=i}^{n} (x_i\ \mu)^2\ p(x_i)}$ is called the standard deviation of the random variable X.

$Var\ (X) = E\left(X^2\right) - \left[E(X)\right]^2$

- Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

(i) There should be a finite number of trials.

(ii) The trials should be independent.

(iii) Each trial has exactly two outcomes: success or failure.

(iv) The probability of success remains the same in each trial.

For Binomial distribution $B(n, p),\ P(X{=}x) = {}^n C_x q^{n-x} P^x, x = 0, 1, \ldots, n(q = 1 - p)$