FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Thursday 01st February, 2024) **TIME : 3 : 00 PM to 06 : 00 PM**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let $f(x) = |2x^2+5|x|-3|, x\in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then m + n is equal to :

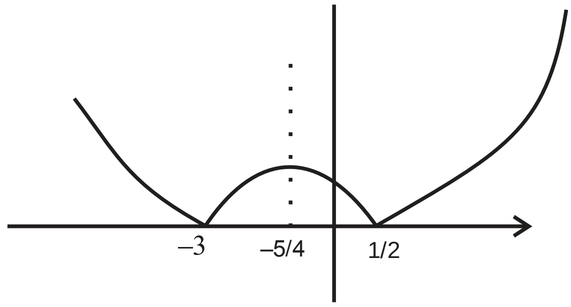
(1) 5 (2) 2

(3) 0 (4) 3

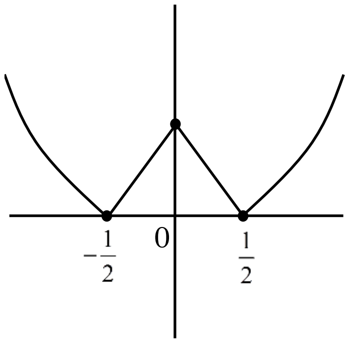
Ans. (4)

Sol. $f(x) = |2x^2+5|x|-3|$

Graph of $y = |2x^2 + 5x - 3|$



Graph of f(x)



Number of points of discontinuity = 0 = m

Number of points of non-differentiability = 3 = n

2. Let α and β be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non-constant G.P and $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$, then the value of $(\alpha - \beta)^2$ is :

(1) $\frac{80}{9}$ (2) 9

(3) $\frac{20}{3}$ (4) 8

Ans. (1)

Sol. $px^2 + qx - r = 0 <^{\alpha}_{\beta}$

$p = A, q = AR, r = AR^2$

$Ax^2 + ARx - AR^2 = 0$

$x^2 + Rx - R^2 = 0 <^{\alpha}_{\beta}$

$\because \frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$

$\therefore \frac{\alpha+\beta}{\alpha\beta}=\frac{3}{4} \Rightarrow \frac{-R}{-R^2}=\frac{3}{4} \Rightarrow R=\frac{4}{3}$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = R^2 - 4(-R^2) = 5\left(\frac{16}{9}\right)$

$= 80/9$

3. The number of solutions of the equation $4\sin^2x - 4\cos^3x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is :

(1) 1

(2) 3

(3) 2

(4) 0

Ans. (4)

Sol. $4\sin^2x - 4\cos^3x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$

$4 - 4\cos^2x - 4\cos^3x + 9 - 4\cos x = 0$

$4\cos^3x + 4\cos^2x + 4\cos x - 13 = 0$

$4\cos^3x + 4\cos^2x + 4\cos x = 13$

L.H.S. ≤ 12 can't be equal to 13.

4. The value of $\int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}}\,dx$ is equal to:

(1) 0

(2) 1

(3) 2

(4) –1

Ans. (1)

Sol. $I = \int_0^1 \left(2x^3 - 3x^2 - x + 1\right)^{\frac{1}{3}} dx$

Using $\int_0^{2a} f(x)dx$ where $f(2a-x) = -f(x)$

Here $f(1-x) = f(x)$

$\therefore I = 0$

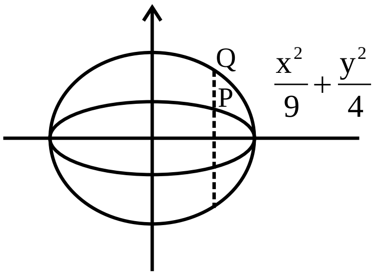
5. Let P be a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that PR : RQ = 4 : 3 as P moves on the ellipse, is :

(1) $\frac{11}{19}$ (2) $\frac{13}{21}$

(3) $\frac{\sqrt{139}}{23}$ (4) $\frac{\sqrt{13}}{7}$

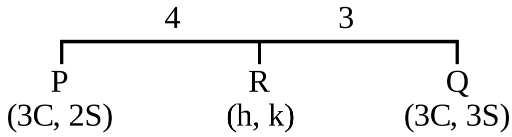
Ans. (4)

Sol.



$\frac{x^2}{9}+\frac{y^2}{4}=1$

P(3cosθ, 2sinθ)
Q(3cosθ, 3sinθ)



$h = 3\cos\theta$;

$k=\frac{18}{7}\sin\theta$

$\therefore$ locus $=\frac{x^2}{9}+\frac{49y^2}{324}=1$

$e=\sqrt{1-\frac{324}{49\times 9}}=\frac{\sqrt{117}}{21}=\frac{\sqrt{13}}{7}$

6. Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}}+\frac{1}{2x^{\frac{2}{3}}}\right)^{18}$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is :

(1) $\frac{4}{9}$ (2) $\frac{1}{9}$

(3) $\frac{1}{4}$ (4) $\frac{9}{4}$

Ans. (4)

Sol. $\left(\frac{x^{\frac{1}{3}}}{3}+\frac{x^{\frac{-2}{3}}}{\ }\right)^{18}$

$t_7 = {}^{18}c_6\left(\frac{x^{\frac{1}{3}}}{3}\right)^{12}\left(\frac{x^{\frac{-2}{3}}}{2}\right)^{6} = {}^{18}c_6\frac{1}{(3)^{12}}.\frac{1}{2^6}$

$t_{13} = {}^{18}c_{12}\left(\frac{x^{\frac{1}{3}}}{3}\right)^{6}\left(\frac{x^{\frac{-2}{3}}}{2}\right)^{12} = {}^{18}c_{12}\frac{1}{(3)^{6}}.\frac{1}{2^{12}}.x^{-6}$

$m = {}^{18}c_6.3^{-12}.2^{-6}\ :\ n = {}^{18}c_{12}.2^{-12}.3^{-6}$

$\left(\frac{n}{m}\right)^{\frac{1}{3}}=\left(\frac{2^{-12}.3^{-6}}{3^{-12}.2^{-6}}\right)^{\frac{1}{3}}=\left(\frac{3}{2}\right)^{2}=\frac{9}{4}$

7. Let α be a non-zero real number. Suppose f : R → R is a differentiable function such that $f(0) = 2$ and $\lim_{x\to-\infty} f(x)=1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in R$, then $f(-\log_e 2)$ is equal to____.

(1) 3 (2) 5

(3) 9 (4) 7

Ans. (3 OR BONUS)

Sol. $f(0) = 2,\ \lim_{x\to-\infty} f(x)=1$

$f'(x) - x.f(x) = 3$

$I.F = e^{-\alpha x}$

$y(e^{-\alpha x}) = \int 3.e^{-\alpha x}dx$

$f(x).\ (e^{-\alpha x}) = \frac{3e^{-\alpha x}}{-\alpha}+c$

$x = 0 \Rightarrow 2=\frac{-3}{\alpha}+c \Rightarrow \frac{3}{\alpha}=c-2$ (1)

$f(x) = \frac{-3}{\alpha}+c.e^{\alpha x}$

$x \to -\infty \Rightarrow 1 = \frac{-3}{\alpha}+c(0)$

$\alpha = -3 \therefore c = 1$

$f(-\ln 2)=\frac{-3}{\alpha}+c.e^{\alpha x}$

$= 1 + e^{3\ln 2} = 9$

(But α should be greater than 0 for finite value of c)

8. Let P and Q be the points on the line $\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$ which are at a distance of 6 units from the point R (1,2,3). If the centroid of the triangle PQR is (α, β, γ), then $\alpha^2+\beta^2+\gamma^2$ is:

(1) 26

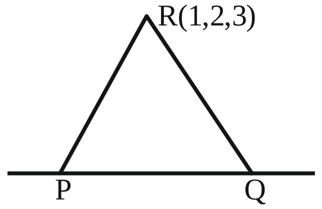
(2) 36

(3) 18

(4) 24

Ans. **(3)**

Sol.



$P(8\lambda-3, 2\lambda+4, 2\lambda-1)$

PR = 6

$(8\lambda-4)^2+(2\lambda+2)^2+(2\lambda-4)^2=36$

$\lambda = 0, 1$

Hence P(–3, 4, –1) & Q(5, 6, 1)

Centroid of $\Delta PQR = (1, 4, 1) \equiv (\alpha, \beta, \gamma)$

$\alpha^2+\beta^2+\gamma^2 = 18$

9. Consider a ΔABC where A(1,2,3,), B(–2,8,0) and C(3,6,7). If the angle bisector of $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\vec{AD}$ on the vector $\vec{AC}$ is:

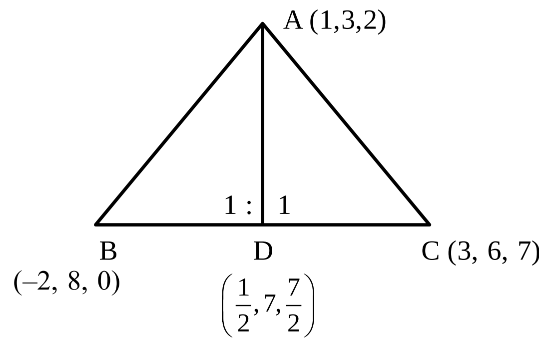
(1) $\frac{37}{2\sqrt{38}}$

(2) $\frac{\sqrt{38}}{2}$

(3) $\frac{39}{2\sqrt{38}}$

(4) $\sqrt{19}$

Ans. **(1)**



Sol.

A(1, 3, 2); B(–2, 8, 0); C(3, 6, 7);

$\overrightarrow{AC}=2\hat{i}+3\hat{j}+5\hat{k}$

AB = $\sqrt{9+25+4}=\sqrt{38}$

AC = $\sqrt{4+9-25}=\sqrt{38}$

$\overrightarrow{AD}=\frac{1}{2}\hat{i}-4\hat{j}-\frac{3}{2}\hat{k}=\frac{1}{2}(\hat{i}-8\hat{j}-3\hat{k})$

Length of projection of $\overrightarrow{AD}$ on $\overrightarrow{AC}$

$=\left|\frac{\overrightarrow{AD}.\overrightarrow{AC}}{|\overrightarrow{AC}|}\right|=\frac{37}{2\sqrt{38}}$

10. Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to:

(1) 800

(2) 890

(3) 790

(4) 690

Ans. **(3)**

Sol. $S_{10} = 390$

$\frac{10}{2}\left[2a+(10-1)d\right]=390$

$\Rightarrow 2a + 9d = 78$ (1)

$\frac{t_{10}}{t_5}=\frac{15}{7}\Rightarrow\frac{a+9d}{a+4d}=\frac{15}{7}\Rightarrow 8a=3d$ (2)

From (1) & (2) a = 3 & d = 8

$S_{15}-S_5=\frac{15}{2}(6+14\times8)-\frac{5}{2}(6+4\times8)$

$=\frac{15\times118-5\times38}{2}$ =790

11. If $\int_0^{\frac{\pi}{3}} \cos^4 x\, dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then 9a + 8b is equal to :

(1) 2 (2) 1

(3) 3 (4) $\frac{3}{2}$

Ans. (1)

Sol. $\int_0^{\pi/3} \cos^4 x dx$

$= \int_0^{\pi/3} \left(\frac{1+\cos 2x}{2}\right)^2 dx$

$= \frac{1}{4}\int_0^{\pi/3} (1+2\cos 2x + \cos^2 2x)dx$

$= \frac{1}{4}\left[\int_0^{\pi/3} dx + 2\int_0^{\pi/3} \cos 2x\, dx + \int_0^{\pi/3} \frac{1+\cos 4x}{2} dx\right]$

$= \frac{1}{4}\left[\frac{\pi}{3} + (\sin 2x)_0^{\pi/3} + \frac{1}{2}\left(\frac{\pi}{3}\right) + \frac{1}{8}(\sin 4x)_0^{\pi/3}\right]$

$= \frac{1}{4}\left[\frac{\pi}{3} + (\sin 2x)_0^{\pi/3} + \frac{1}{2}\left(\frac{\pi}{3}\right) + \frac{1}{8}(\sin 4x)_0^{\pi/3}\right]$

$= \frac{1}{4}\left[\frac{\pi}{2} + \frac{\sqrt{3}}{2} + \frac{1}{8} \times \left(-\frac{\sqrt{3}}{2}\right)\right]$

$= \frac{\pi}{2} + \frac{7\sqrt{3}}{64}$

$\therefore a = \frac{1}{8}; b = \frac{7}{64}$

$\therefore 9a + 8b = \frac{9}{8} + \frac{7}{8} = 2$

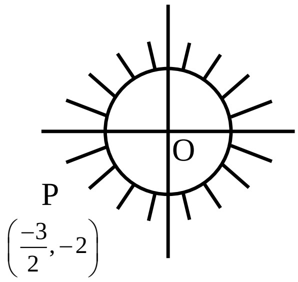
12. If z is a complex number such that $|z| \geq 1$, then the minimum value of $\left|z + \frac{1}{2}(3+4i)\right|$ is:

(1) $\frac{5}{2}$ (2) 2

(3) 3 (4) $\frac{3}{2}$

Ans. (Bonus)

Sol. $|z| \geq 1$



Min. value of $\left|z + \frac{3}{2} + 2i\right|$ is actually zero.

13. If the domain of the function $f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)}$ $+\log_{10}(x^2 + 2x - 15)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to :

(1) 140 (2) 175

(3) 150 (4) 125

Ans. (3)

Sol. $f(x) = \frac{\sqrt{x^2-25}}{4-x^2} + \log_{10}(x^2 + 2x - 15)$

Domain : $x^2 - 25 \geq 0 \Rightarrow x \in (-\infty, -5] \cup [5, \infty)$

$4 - x^2 \neq 0 \Rightarrow x \neq \{-2, 2\}$

$x^2 + 2x - 15 > 0 \Rightarrow (x + 5)(x - 3) > 0$

$\Rightarrow x \in (-\infty, -5) \cup (3, \infty)$

$\therefore x \in (-\infty, -5) \cup [5, \infty)$

$\alpha = -5; \beta = 5$

$\therefore \alpha^2 + \beta^3 = 150$

14. Consider the relations R_1 and R_2 defined as $aR_1b \Leftrightarrow a^2 + b^2 = 1$ for all a , b, $\in$ R and (a, b) R_2(c, d) $\Leftrightarrow a + d = b + c$ for all (a,b), (c,d) $\in N \times N$. Then

(1) Only R_1 is an equivalence relation

(2) Only R_2 is an equivalence relation

(3) R_1 and R_2 both are equivalence relations

(4) Neither R_1 nor R_2 is an equivalence relation

Ans. (2)

Sol. $aR_1\, b \Leftrightarrow a^2 + b^2 = 1; a, b \in R$

$(a, b)\, R_2\, (c, d) \Leftrightarrow a + d = b + c; (a, b), (c, d) \in N$

for R_1 : Not reflexive symmetric not transitive

for R_2 : R_2 is reflexive, symmetric and transitive

Hence only R_2 is equivalence relation.

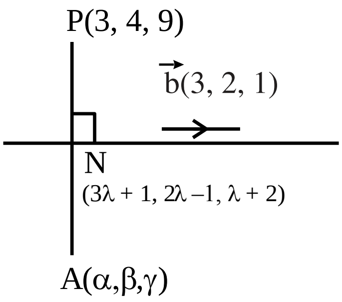
15. If the mirror image of the point P(3,4,9) in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α, β, γ), then $14(\alpha+\beta+\gamma)$ is :

(1) 102 (2) 138

(3) 108 (4) 132

Ans. (3)

Sol.



$\overrightarrow{PN}.\vec{b}=0$?

$3(3\lambda-2)+2(2\lambda-5)+(\lambda-7)=0$

$14\lambda=23 \Rightarrow \lambda=\frac{23}{14}$

$N\left(\frac{83}{14},\frac{32}{14},\frac{51}{14}\right)$

$\therefore \frac{\alpha+3}{2}=\frac{83}{14} \Rightarrow \alpha=\frac{62}{7}$

$\frac{\beta+4}{2}=\frac{32}{14} \Rightarrow \beta=\frac{4}{7}$

$\frac{\gamma+9}{2}=\frac{51}{14} \Rightarrow \gamma=\frac{-12}{7}$

Ans. $14(\alpha+\beta+r)=108$

16. Let $f(x)=\begin{cases} x-1, x \text{ is even,} \\ 2x, \quad x \text{ is odd,} \end{cases}$ $x \in N$. If for some $a \in N$, $f(f(f(a)))=21$, then $\lim_{x\to a^-}\left\{\frac{|x|^3}{a}-\left[\frac{x}{a}\right]\right\}$, where [t] denotes the greatest integer less than or equal to t, is equal to :

(1) 121

(2) 144

(3) 169

(4) 225

Ans. (2)

Sol. $f(x)=\begin{cases} x-1; & x=\text{even} \\ 2x; & x=\text{odd} \end{cases}$

$f(f(f(a)))=21$

C–1: If a = even

$f(a)=a-1=\text{odd}$

$f(f(a))=2(a-1)=\text{even}$

$f(f(f(a)))=2a-3=21 \Rightarrow a=12$

C–2: If a = odd

$f(a)=2a=\text{even}$

$f(f(a))=2a-1=\text{odd}$

$f(f(f(a)))=4a-2=21$ (Not possible)

Hence a = 12

Now

$$\lim_{x\to 12^-}\left(\frac{|x|^3}{2}-\left[\frac{x}{12}\right]\right)$$

$$=\lim_{x\to 12^-}\frac{|x|^3}{12}-\lim_{x\to 12^-}\left[\frac{x}{12}\right]$$

= 144 – 0 = 144.

17. Let the system of equations $x+2y+3z=5$, $2x+3y+z=9$, $4x+3y+\lambda z=\mu$ have infinite number of solutions. Then $\lambda+2\mu$ is equal to :

(1) 28 (2) 17

(3) 22 (4) 15

Ans. (2)

Sol. $x+2y+3z=5$

$2x+3y+z=9$

$4x+3y+\lambda z=\mu$

for infinite following $\Delta=\Delta_1=\Delta_2=\Delta_3=0$

$$\Delta=\begin{vmatrix}1&2&3\\2&3&1\\4&3&\lambda\end{vmatrix}=0 \Rightarrow \lambda=-13$$

$$\Delta_1=\begin{vmatrix}5&2&3\\9&3&1\\\mu&3&-13\end{vmatrix}=0 \Rightarrow \mu=15$$

$$\Delta_2=\begin{vmatrix}1&5&3\\2&9&1\\4&15&-13\end{vmatrix}=0$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & 15 \end{vmatrix} = 0$$

for $\lambda = -13$, $\mu = 15$ system of equation has infinite solution hence $\lambda + 2\mu = 17$

18. Consider 10 observation $x_1, x_2, \ldots, x_{10}$. such that $\sum_{i=1}^{10} (x_i - \alpha) = 2$ and $\sum_{i=1}^{10} (x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. The $\frac{\beta}{\alpha}$ is equal to :

(1) 2　　(2) $\frac{3}{2}$

(3) $\frac{5}{2}$　　(4) 1

Ans. **(1)**

Sol. $x_1, x_2 \ldots\ldots x_{10}$

$$\sum_{i=1}^{10}(x_i - \alpha) = 2 \Rightarrow \sum_{i=1}^{10} x_i - 10\alpha = 2$$

$$\text{Mean } \mu = \frac{6}{5} = \frac{\sum x_i}{10}$$

$\therefore \ \Sigma x_i = 12$

$10\alpha + 2 = 12 \ \therefore \alpha = 1$

Now $\sum_{i=1}^{10}(x_i - \beta)^2 = 40$　Let $y_i = x_i - \beta$

$$\therefore \sigma_y^2 = \frac{1}{10}\sum y_i^2 - (\overline{y})^2$$

$$\sigma_x^2 = \frac{1}{10}\sum (x_i - \beta)^2 - \left(\frac{\sum_{i=1}^{10}(x_i - \beta)}{10}\right)^2$$

$$\frac{84}{25} = 4 - \left(\frac{12 - 10\beta}{10}\right)^2$$

$$\therefore \left(\frac{6 - 5\beta}{5}\right)^2 = 4 - \frac{84}{25} = \frac{16}{25}$$

$6 - 5\beta = \pm 4 \Rightarrow \beta = \frac{2}{5}$ (not possible) or $\beta = 2$

Hence $\frac{\beta}{\alpha} = 2$

19. Let Ajay will not appear in JEE exam with probability $p = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q = \frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is :

(1) $\frac{9}{35}$

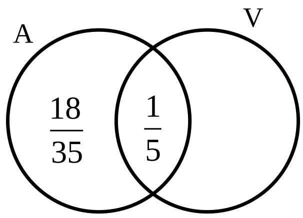
(2) $\frac{18}{35}$

(3) $\frac{24}{35}$

(4) $\frac{3}{35}$

Ans. (2)

Sol.



$$P(\overline{A}) = \frac{2}{7} = p$$

$$P(A \cap V) = \frac{1}{5} = q$$

$$P(A) = \frac{5}{7}$$

Ans. $P(A \cap \overline{V}) = \frac{18}{35}$

20. Let the locus of the mid points of the chords of circle $x^2 + (y-1)^2 = 1$ drawn from the origin intersect the line $x + y = 1$ at P and Q. Then, the length of PQ is :

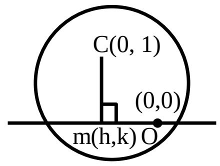
(1) $\frac{1}{\sqrt{2}}$

(2) $\sqrt{2}$

(3) $\frac{1}{2}$

(4) 1

Ans. **(1)**

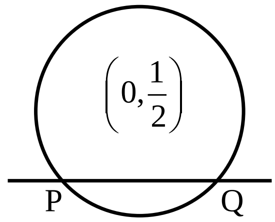
Sol. 

$m_{OM} \cdot m_{CM} = -1$

$\frac{k}{h} \cdot \frac{k-1}{h} = -1$

$\therefore$ locus is $x^2 + y(y-1) = 0$

$x^2 + y^2 - y = 0$



$p = \left|\frac{1/2}{\sqrt{2}}\right| \qquad p = \frac{1}{2\sqrt{2}}$

$PQ = 2\sqrt{r^2 - p^2}$

$= 2\sqrt{\frac{1}{4} - \frac{1}{8}} = \frac{1}{\sqrt{2}}$

SECTION-B

21. If three successive terms of a G.P. with common ratio r(r > 1) are the lengths of the sides of a triangle and [r] denotes the greatest integer less than or equal to r, then 3[r] + [–r] is equal to :

Ans. (1)

Sol. a, ar, $ar^2 \rightarrow$ G.P.

Sum of any two sides > third side

$a + ar > ar^2$, $a + ar^2 > ar$, $ar + ar^2 > a$

$r^2 - r - 1 < 0$

$r \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$ (1)

$r^2 - r + 1 > 0$

always true

$r^2 + r - 1 > 0$

$r \in \left(-\infty, -\frac{1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right)$ (2)

Taking intersection of (1) , (2)

$r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$

As $r > 1$

$r \in \left(1, \frac{1+\sqrt{5}}{2}\right)$

$[r] = 1 \quad [-r] = -2$

$3[r] + [-r] = 1$

22. Let $A = I_2 - MM^T$, where M is real matrix of order 2 × 1 such that the relation $M^T M = I_1$ holds. If λ is a real number such that the relation AX = λX holds for some non-zero real matrix X of order 2 × 1, then the sum of squares of all possible values of λ is equal to :

Ans. (2)

Sol. $A = I_2 - 2MM^T$

$A^2 = (I_2 - 2MM^T)(I_2 - 2MM^T)$

$= I_2 - 2MM^T - 2MM^T + 4MM^TMM^T$

$= I_2 - 4MM^T + 4MM^T$

$= I_2$

$AX = \lambda X$

$A^2X = \lambda AX$

$X = \lambda(\lambda X)$

$X = \lambda^2 X$

$X(\lambda^2 - 1) = 0$

$\lambda^2 = 1$

$\lambda = \pm 1$

Sum of square of all possible values = 2

23. Let f : (0, ∞) → R and $F(x)=\int_0^x tf(t)dt$. If $F(x^2)$ = $x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2)$ is equal to :

Ans. (219)

Sol. $F(x) = \int_0^x t\cdot f(t)dt$

$F^1(x) = xf(x)$

Given $F(x^2) = x^4 + x^5$, let $x^2 = t$

$F(t) = t^2 + t^{5/2}$

$F'(t) = 2t + 5/2\ t^{3/2}$

$t\cdot f(t) = 2t + 5/2\ t^{3/2}$

$f(t) = 2 + 5/2\ r^{1/2}$

$\sum_{r=1}^{12} f(r^2) = \sum_{r=1}^{12} 2 + \frac{5}{2}r$

$= 24 + 5/2\left[\frac{12(13)}{2}\right]$

$=219$

24. If $y=\frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}}+\frac{1}{15}(3\cos^2 x-5)\cos^3 x$, then $96y'\left(\frac{\pi}{6}\right)$ is equal to :

Ans. (105)

Sol. $y=\frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}}+\frac{1}{15}(3\cos^2x-5)\cos^3x$

$y=\frac{(\sqrt{x}+1)(\sqrt{x})\left((\sqrt{x})^3-1\right)}{(\sqrt{x})\left((\sqrt{x})^2+(\sqrt{x})+1\right)}+\frac{1}{5}\cos^5x-\frac{1}{3}\cos^3x$

$y=(\sqrt{x}+1)\ (\sqrt{x}-1)+\frac{1}{5}\ \cos^5x-\frac{1}{3}\cos^3x$

$y' = 1 - \cos^4x\cdot(\sin x) + \cos^2x\ (\sin x)$

$y'\left(\frac{\pi}{6}\right)=1-\frac{9}{16}\times\frac{1}{2}+\frac{3}{4}\times\frac{1}{2}$

$=\frac{32-9+12}{32}\ =\frac{35}{32}$

$= 96\ y'\left(\frac{\pi}{6}\right) = 105$

25. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=-\hat{i}-8\hat{j}+2\hat{k}$ and $\vec{c}=4\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three vectors such that $\vec{b}\times\vec{a}=\vec{c}\times\vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i}+4\hat{j}+\hat{k}$ is θ, then the greatest integer less than or equal to $\tan^2\theta$ is :

Ans. (38)

Sol. $\vec{a}=\hat{i}+\hat{j}+k$

$\vec{b}=\hat{i}+8\hat{j}+2k$

$\vec{c}=4\hat{i}+c_2\hat{j}+c_3k$

$\vec{b}\times\vec{a}=\vec{c}\times\vec{a}$

$(\vec{b}-\vec{c})\times\vec{a}=0$

$\vec{b}-\vec{c}=\lambda\vec{\alpha}$

$\vec{b}=\vec{c}+\lambda\vec{\alpha}$

$-\hat{i}-8\hat{j}+2k=(4\hat{i}+c_2\hat{j}+c_3k)+\lambda(\hat{i}+\hat{j}+k)$

$\lambda + 4 = -1 \Rightarrow \lambda = -5$

$\lambda + c_2 = -8 \Rightarrow c_2 = -3$

$\lambda + c_3 = 2 \Rightarrow c_3 = 7$

$\vec{c}=4\hat{i}-3\hat{j}+7k$

$\cos\theta=\frac{12-12+7}{\sqrt{26}\cdot\sqrt{74}}=\frac{7}{\sqrt{26}\cdot\sqrt{74}}=\frac{7}{2\sqrt{481}}$

$\tan^2\theta=\frac{625\times3}{49}$

$[\tan^2\theta] = 38$

26. The lines L_1, L_2,…., I_{20} are distinct. For n = 1, 2, 3,…., 10 all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2,…., L_{20}\}$ is equal to :

Ans. (101)

Sol. $L_1, L_3, L_5, - - L_{19}$ are Parallel

$L_2, L_4, L_6, - - L_{20}$ are Concurrent

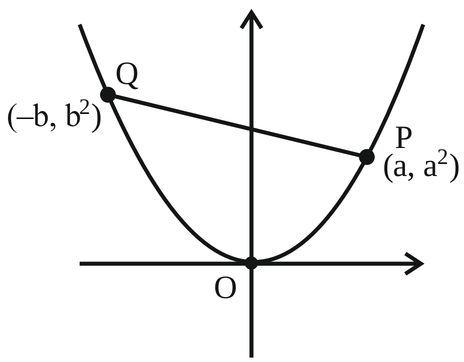
Total points of intersection = ${}^{20}C_2 - {}^{10}C_2 - {}^{10}C_2 + 1$

= 101

27. Three points O(0,0), P(a, a^2), Q(–b, b^2), a > 0, b > 0, are on the parabola $y = x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ. If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}$, gcd(m, n) = 1, then m + n is equal to :

Ans. (7)

Sol.



$$S_2 = 1/2 \left| \begin{vmatrix} 0 & 0 & 1 \\ a & a^2 & 1 \\ -b & b^2 & 1 \end{vmatrix} \right| = 1/2\left(ab^2 + a^2b\right)$$

PQ:- $y - a^2 = \frac{a^2 - b^2}{a+b}(x - a)$

$y - a^2 = (a - b)\,x - (a - b)a$

$y = (a - b)\,x + ab$

$$S_1 = \int_{-b}^{a}\left((a-b)x + ab - x^2\right)dx$$

$$= (a-b)\frac{x^2}{2} + (ab)x - \frac{x^3}{3}\Big|_{-b}^{a}$$

$$= \frac{(a-b)^2(a+b)}{2} + ab(a+b) - \frac{(a^3+b^3)}{3}$$

$$\frac{S_1}{S_2} = \frac{\frac{(a-b)^2}{2} + ab - \frac{(a^2+b^2-ab)}{3}}{\frac{ab}{2}}$$

$$= \frac{3(a-b)^2 + 6ab - 2(a^2+b^2-ab)}{3ab}$$

$$= \frac{1}{3}\left[\underbrace{\frac{a}{b} + \frac{b}{a}}_{\min=2} + 2\right]$$

$$= \frac{4}{3} = \frac{m}{n} \qquad m + n = 7$$

28. The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y - x)$ is maximum, is equal to :

Ans. (8)

Sol. $ky^2 = 2(y - x)$ $\quad\quad 2y^2 = kx$

Point of intersection →

$$ky^2 = \left(y - \frac{2y^2}{k}\right)$$

$y = 0$ $\quad\quad ky = 2\left(1 - \frac{2y}{k}\right)$

$$ky + \frac{4y}{k} = 2$$

$$y = \frac{2}{k + \frac{4}{k}} = \frac{2k}{k^2 + 4}$$

$$A = \int_0^{\frac{2k}{k^2+4}} \left(\left(y - \frac{ky^2}{2}\right) - \left(\frac{2y^2}{k}\right)\right).dy$$

$$= \frac{y^2}{2} - \left(\frac{k}{2} + \frac{2}{k}\right) \cdot \frac{y^3}{3}\Bigg|_0^{\frac{2k}{k^2+4}}$$

$$= \left(\frac{2k}{k^2 + 4}\right)^2 \left[\frac{1}{2} - \frac{k^2 + 4}{2k} \times \frac{1}{3} \times \frac{2k}{k^2 + 4}\right]$$

$$= \frac{1}{6} \times 4 \times \left(\frac{1}{k + \frac{4}{k}}\right)^2$$

$A \cdot M \geq G \cdot M \quad \frac{\left(k + \frac{4}{k}\right)}{2} \geq 2$

$$k + \frac{4}{k} \geq 4$$

Area is maximum when $k = \frac{4}{k}$

$k = 2, -2$

29. If $\frac{dx}{dy} = \frac{1 + x - y^2}{y}$, $x(1) = 1$, then $5x(2)$ is equal to :

Ans. **(5)**

Sol. $\frac{dx}{dy} - \frac{x}{y} = \frac{1 - y^2}{y}$

Integrating factor $= e^{\int -\frac{1}{y} dy} = \frac{1}{y}$

$x \cdot \frac{1}{y} = \int \frac{1 - y^2}{y^2} dy$

$\frac{x}{y} = \frac{-1}{y} - y + c$

$x = -1 - y^2 + cy$

$x(1) = 1$

$1 = -1 - 1 + c \Rightarrow c = 3$

$x = -1 - y^2 + 3y$

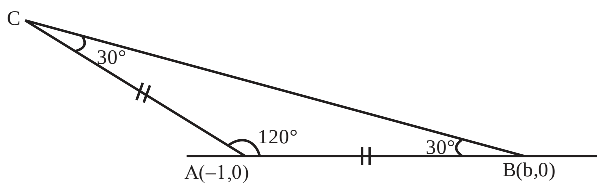
$5x(2) = 5(-1 - 4 + 6)$

$= 5$

30. Let ABC be an isosceles triangle in which A is at $(-1, 0)$, $\angle A = \frac{2\pi}{3}$, AB = AC and B is on the positive x-axis. If $BC = 4\sqrt{3}$ and the line BC intersects the line $y = x + 3$ at (α, β), then $\frac{\beta^4}{\alpha^2}$ is :

Ans. **(36)**

Sol.



$\frac{c}{\sin 30°} = \frac{4\sqrt{3}}{\sin 120°}$ [By sine rule]

$2c = 8 \Rightarrow c = 4$

$AB = |(b+1)| = 4$

$b = 3,\ m_{AB} = 0$

$m_{BC} = \frac{-1}{\sqrt{3}}$

BC:- $y = \frac{-1}{\sqrt{3}}(x-3)$

$\sqrt{3}y + x = 3$

Point of intersection : $y = x + 3,\ \sqrt{3}y + x = 3$

$$\left(\sqrt{3}+1\right)y = 6$$

$$y = \frac{6}{\sqrt{3}+1}$$

$$x = \frac{6}{\sqrt{3}+1} - 3$$

$$= \frac{6 - 3\sqrt{3} - 3}{\sqrt{3}+1}$$

$$= 3\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)} = \frac{-6}{\left(1+\sqrt{3}\right)^2}$$

$$\frac{\beta^4}{\alpha^2} = 36$$

SECTION-A

31. In an ammeter, 5% of the main current passes through the galvanometer. If resistance of the galvanometer is G, the resistance of ammeter will be :

(1) $\frac{G}{200}$

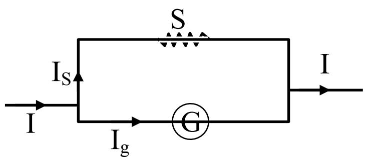
(2) $\frac{G}{199}$

(3) 199 G

(4) 200 G

Ans. **(Bonus)**

Sol.



$I_S S = I_g G$

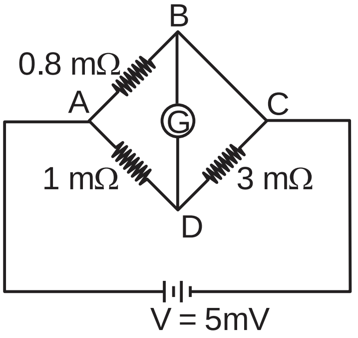
$$\frac{95}{100} IS = \frac{5I}{100} G$$

$$S = \frac{G}{19}$$

$$R_A = \frac{SG}{S+G} = \frac{\frac{G^2}{19}}{\frac{20G}{19}}$$

$$R_A = \frac{G}{20}$$

32. To measure the temperature coefficient of resistivity α of a semiconductor, an electrical arrangement shown in the figure is prepared. The arm BC is made up of the semiconductor. The experiment is being conducted at 25°C and resistance of the semiconductor arm is 3 mΩ. Arm BC is cooled at a constant rate of 2°C/s. If the galvanometer G shows no deflection after 10s, then α is :



(1) $-2 \times 10^{-2}\,°C^{-1}$

(2) $-1.5 \times 10^{-2}\,°C^{-1}$

(3) $-1 \times 10^{-2}\,°C^{-1}$

(4) $-2.5 \times 10^{-2}\,°C^{-1}$

Ans. **(3)**

Sol. For no deflection $\frac{0.8}{1} = \frac{R}{3}$

$\Rightarrow R = 2.4 m\Omega$

Temperature fall in 10s = 20°C

$\Delta R = R\,\alpha\,\Delta t$

$$\alpha = \frac{\Delta R}{R \Delta t} = \frac{-0.6}{3 \times 20}$$

$= -10^{-2} C^{-1}$

33. From the statements given below :

(A) The angular momentum of an electron in n^{th} orbit is an integral multiple of h.

(B) Nuclear forces do not obey inverse square law.

(C) Nuclear forces are spin dependent.

(D) Nuclear forces are central and charge independent.

(E) Stability of nucleus is inversely proportional to the value of packing fraction.

Choose the correct answer from the options given below :

(1) (A), (B), (C), (D) only

(2) (A), (C), (D), (E) only

(3) (A), (B), (C), (E) only

(4) (B), (C), (D), (E) only

Ans. **(3)**

Sol. Part of theory

34. A diatomic gas (γ = 1.4) does 200 J of work when it is expanded isobarically. The heat given to the gas in the process is :

(1) 850 J (2) 800 J

(3) 600 J (4) 700 J

Ans. **(4)**

Sol. $\gamma = 1 + \frac{2}{f} = 1.4 \Rightarrow \frac{2}{f} = 0.4$

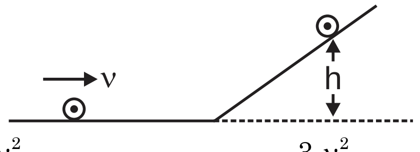
$\Rightarrow f = 5$

$W = n R \Delta T = 200J$

$Q = \left(\frac{f+2}{2}\right) nR\Delta T$

$= \frac{7}{2} \times 200 \quad = 700 \text{ J}$

35. A disc of radius R and mass M is rolling horizontally without slipping with speed ν. It then moves up an inclined smooth surface as shown in figure. The maximum height that the disc can go up the incline is :



(1) $\frac{\nu^2}{g}$ (2) $\frac{3}{4}\frac{\nu^2}{g}$

(3) $\frac{1}{2}\frac{\nu^2}{g}$ (4) $\frac{2}{3}\frac{\nu^2}{g}$

Ans. **(3)**

Sol. Only the translational kinetic energy of disc changes into gravitational potential energy. And rotational KE remains unchanged as there is no friction.

$\frac{1}{2}mv^2 = mgh$

$h = \frac{v^2}{2g}$

36. Conductivity of a photodiode starts changing only if the wavelength of incident light is less than 660 nm. The band gap of photodiode is found to be $\left(\frac{X}{8}\right)$eV . The value of X is :

(Given, $h = 6.6 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$C)

(1) 15 (2) 11

(3) 13 (4) 21

Ans. **(1)**

Sol. $E_g = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{660\times10^{-9}} \text{ J}$

$= \frac{6.6\times10^{-34}\times3\times10^{8}}{660\times10^{-9}\times1.6\times10^{-19}} \text{eV}$

$= \frac{15}{8} \text{eV}$

So x = 15

37. A big drop is formed by coalescing 1000 small droplets of water. The surface energy will become :

(1) 100 times (2) 10 times

(3) $\frac{1}{100}$th (4) $\frac{1}{10}$th

Ans. **(4)**

Sol. Lets say radius of small droplets is r and that of big drop is R

$\frac{4}{3}\pi R^3 = 1000\frac{4}{3}\pi r^3$

R = 10r

$U_i = 1000\,(4\pi r^2 S)$

$U_f = 4\pi R^2 S$

$= 100\,(4\pi r^2 S)$

$U_f = \frac{1}{10} U_i$

38. If frequency of electromagnetic wave is 60 MHz and it travels in air along z direction then the corresponding electric and magnetic field vectors will be mutually perpendicular to each other and the wavelength of the wave (in m) is :

(1) 2.5 (2) 10

(3) 5 (4) 2

Ans. (3)

Sol. $\lambda = \frac{c}{f} = \frac{3\times10^8}{60\times10^6} = 5\text{m}$

39. A cricket player catches a ball of mass 120 g moving with 25 m/s speed. If the catching process is completed in 0.1 s then the magnitude of force exerted by the ball on the hand of player will be (in SI unit):

(1) 24 (2) 12

(3) 25 (4) 30

Ans. (4)

Sol. $F_{av} = \frac{\Delta p}{\Delta t}$

$= \frac{0.12\times25}{0.1} = 30\text{N}$

40. Monochromatic light of frequency 6×10^{14} Hz is produced by a laser. The power emitted is 2×10^{-3} W. How many photons per second on an average, are emitted by the source ?

(Given $h = 6.63 \times 10^{-34}$ Js)

(1) 9×10^{18} (2) 6×10^{15}

(3) 5×10^{15} (4) 7×10^{16}

Ans. (3)

Sol. $P = nh\nu$

$n = \frac{P}{h\nu} = \frac{2\times10^{-3}}{6.63\times10^{-34}\times6\times10^{14}}$

$= 5 \times 10^{15}$

41. A microwave of wavelength 2.0 cm falls normally on a slit of width 4.0 cm. The angular spread of the central maxima of the diffraction pattern obtained on a screen 1.5 m away from the slit, will be:

(1) 30° (2) 15°

(3) 60° (4) 45°

Ans. (3)

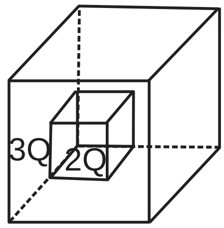
Sol. For first minima $a \sin\theta = \lambda$

$\sin\theta = \frac{\lambda}{a} = \frac{1}{2}$

$\theta = 30°$

Angular spread = 60°

42. C_1 and C_2 are two hollow concentric cubes enclosing charges 2Q and 3Q respectively as shown in figure. The ratio of electric flux passing through C_1 and C_2 is :



(1) 2 : 5 (2) 5 : 2

(3) 2 : 3 (4) 3 : 2

Ans. (1)

Sol. $\phi_{\text{smaller cube}} = \frac{2Q}{\epsilon_0}$

$\phi_{\text{bigger cube}} = \frac{5Q}{\epsilon_0}$

$\frac{\phi_{\text{smaller cube}}}{\phi_{\text{bigger cube}}} = \frac{2}{5}$

43. If the root mean square velocity of hydrogen molecule at a given temperature and pressure is 2 km/s, the root mean square velocity of oxygen at the same condition in km/s is :

(1) 2.0 (2) 0.5

(3) 1.5 (4) 1.0

Ans. (2)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\frac{V_1}{V_2} = \sqrt{\frac{M_2}{M_1}} \Rightarrow \frac{2}{V_2} = \sqrt{\frac{32}{2}}$

$V_2 = 0.5$ km/s

44. Train A is moving along two parallel rail tracks towards north with speed 72 km/h and train B is moving towards south with speed 108 km/h. Velocity of train B with respect to A and velocity of ground with respect to B are (in ms^{-1}) :

(1) –30 and 50

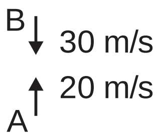
(2) –50 and –30

(3) –50 and 30

(4) 50 and –30

Ans. **(3)**

Sol.



$V_A = 20$ m/s

$V_B = -30$ m/s

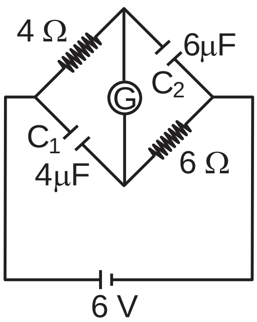
Velocity of B w.r.t. A

$V_{B/A} = -50$ m/s

Velocity of ground w.r.t. B

$V_{G/B} = 30$ m/s

45. A galvanometer (G) of 2Ω resistance is connected in the given circuit. The ratio of charge stored in C_1 and C_2 is :



(1) $\frac{2}{3}$

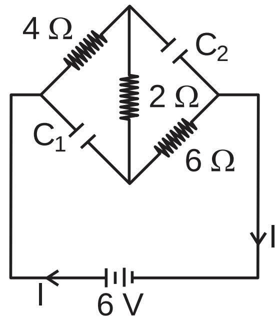
(2) $\frac{3}{2}$

(3) 1

(4) $\frac{1}{2}$

Ans. **(4)**

Sol.



In steady state

Req = 12Ω

$$I = \frac{6}{12} = 0.5A$$

P.D across $C_1 = 3V$

P.D acoross $C_2 = 4V$

$q_1 = C_1V_1 = 12\ \mu C$

$q_2 = C_2V_2 = 24\ \mu C$

$$\frac{q_1}{q_2} = \frac{1}{2}$$

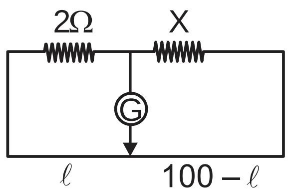
46. In a metre-bridge when a resistance in the left gap is 2Ω and unknown resistance in the right gap, the balance length is found to be 40 cm. On shunting the unknown resistance with 2Ω, the balance length changes by :

(1) 22.5 cm (2) 20 cm

(3) 62.5 cm (4) 65 cm

Ans. **(1)**

Sol.



First case $\frac{2}{40} = \frac{X}{60} \Rightarrow X = 3\Omega$

In second case $X' = \frac{2 \times 3}{2+3} = 1.2\Omega$

$$\frac{2}{\ell} = \frac{1.2}{100 - \ell}$$

$200 - 2\ell = 1.2\ell$

$$\ell = \frac{200}{3.2} = 62.5\text{cm}$$

Balance length changes by 22.5 cm

47. Match List - I with List - II.

List - I	**List - II**
(Number)	**(Significant figure)**
(A) 1001	(I) 3
(B) 010.1	(II) 4
(C) 100.100	(III) 5
(D) 0.0010010	(IV) 6

Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(3) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(4) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

Ans. (3)

Sol. Theoretical

48. A transformer has an efficiency of 80% and works at 10 V and 4 kW. If the secondary voltage is 240 V, then the current in the secondary coil is :

(1) 1.59 A (2) 13.33 A

(3) 1.33 A (4) 15.1 A

Ans. (2)

Sol. Efficiency $= \frac{E_S I_S}{E_P I_P}$

$$0.8 = \frac{240 I_S}{4000}$$

$$I_S = \frac{3200}{240} = 13.33\text{A}$$

49. A light planet is revolving around a massive star in a circular orbit of radius R with a period of revolution T. If the force of attraction between planet and star is proportional to $R^{-3/2}$ then choose the correct option :

(1) $T^2 \propto R^{5/2}$ (2) $T^2 \propto R^{7/2}$

(3) $T^2 \propto R^{3/2}$ (4) $T^2 \propto R^3$

Ans. (1)

Sol. $F = \frac{GMm}{R^{3/2}} = m\omega^2 R$

$\omega^2 \propto \frac{1}{R^{5/2}} \quad \because T = \frac{2\pi}{\omega}$ so

$T^2 \propto R^{5/2}$

50. A body of mass 4 kg experiences two forces $\vec{F}_1 = 5\hat{i} + 8\hat{j} + 7\hat{k}$ and $\vec{F}_2 = 3\hat{i} - 4\hat{j} - 3\hat{k}$. The acceleration acting on the body is :

(1) $-2\hat{i} - \hat{j} - \hat{k}$

(2) $4\hat{i} + 2\hat{j} + 2\hat{k}$

(3) $2\hat{i} + \hat{j} + \hat{k}$

(4) $2\hat{i} + 3\hat{j} + 3\hat{k}$

Ans. (3)

Sol. Net force $= 8\hat{i} + 4\hat{j} + 4\hat{k}$

$$\vec{a} = \frac{\vec{F}}{m} = 2\hat{i} + \hat{j} + \hat{k}$$

SECTION-B

51. A mass m is suspended from a spring of negligible mass and the system oscillates with a frequency f_1. The frequency of oscillations if a mass 9 m is suspended from the same spring is f_2. The value of $\frac{f_1}{f_2}$ is ___.

Ans. (3)

Sol. $f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{9m}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{9}{1}} = \frac{3}{1}$$

52. A particle initially at rest starts moving from reference point. x = 0 along x-axis, with velocity ν that varies as $\nu = 4\sqrt{x}\,\text{m/s}$. The acceleration of the particle is ____ms^{-2}.

Ans. (8)

Sol. $V = 4\sqrt{x}$

$$a = V\frac{dv}{dx}$$

$$= 4\sqrt{x} \times 4 \times \frac{1}{2}x^{-1/2} = 8\ \text{m/s}^2$$

53. A moving coil galvanometer has 100 turns and each turn has an area of 2.0 cm^2. The magnetic field produced by the magnet is 0.01 T and the deflection in the coil is 0.05 radian when a current of 10 mA is passed through it. The torsional constant of the suspension wire is $x \times 10^{-5}$ N-m/rad. The value of x is___.

Ans. **(4)**

Sol. $\tau = BINA\sin\phi$

$C\theta = BINA\sin 90°$

$$C = \frac{BINA}{\theta} = \frac{0.01 \times 10 \times 10^{-3} \times 100 \times 2 \times 10^{-4}}{0.05}$$

$= 4 \times 10^{-5}$ N-m/rad.

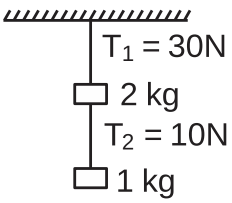
$x = 4$

54. One end of a metal wire is fixed to a ceiling and a load of 2 kg hangs from the other end. A similar wire is attached to the bottom of the load and another load of 1 kg hangs from this lower wire. Then the ratio of longitudinal strain of upper wire to that of the lower wire will be___.

[Area of cross section of wire = 0.005 cm^2, $Y = 2 \times 10^{11}$ Nm^{-2} and $g = 10$ ms^{-2}]

Ans. **(3)**

Sol.



$$\Delta L = \frac{FL}{AY}$$

$$\frac{\Delta L}{L} = \frac{F}{AY}$$

$$\frac{\frac{\Delta L_1}{L_1}}{\frac{\Delta L_2}{L_2}} = \frac{F_1}{F_2} = \frac{30}{10} = 3$$

55. A particular hydrogen - like ion emits the radiation of frequency 3×10^{15} Hz when it makes transition from n = 2 to n = 1. The frequency of radiation emitted in transition from n = 3 to n = 1 is $\frac{x}{9} \times 10^{15}$ Hz, when x = ______.

Ans. **(32)**

Sol. $E = -13.6z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$

$$E = C\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$h\nu = C\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\frac{\nu_1}{\nu_2} = \frac{\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]_{2-1}}{\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]_{3-1}}$$

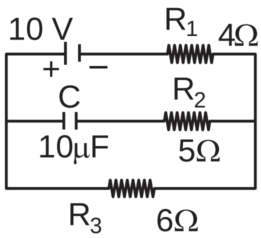
$$= \frac{\left[\frac{1}{1} - \frac{1}{4}\right]}{\left[\frac{1}{1} - \frac{1}{9}\right]} = \frac{3/4}{8/9}$$

$$= \frac{3}{4} \times \frac{9}{8}$$

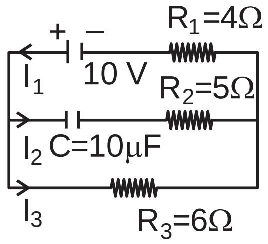
$$\frac{\nu_1}{\nu_2} = \frac{27}{32}$$

$$\nu_2 = \frac{32}{27}\nu_1 = \frac{32}{27} \times 3 \times 10^{15}\,\text{Hz} = \frac{32}{9} \times 10^{15}\,\text{Hz}$$

56. In an electrical circuit drawn below the amount of charge stored in the capacitor is ___μC.



Ans. **(60)**

Sol. 

In steady state there will be no current in branch of capacitor, so no voltage drop across $R_2 = 5\Omega$

$I_2 = 0$

$$I_1 = I_3 = \frac{10}{4+6} = 1A$$

$$V_{R_3} = V_c + V_{R_2} \qquad V_{R_2} = 0$$

$I_3R_3 = V_c$

$V_c = 1 \times 6 = 6$ volt

$q_c = CV_c = 10 \times 6 = 60\ \mu C$

57. A coil of 200 turns and area 0.20 m^2 is rotated at half a revolution per second and is placed in uniform magnetic field of 0.01 T perpendicular to axis of rotation of the coil. The maximum voltage generated in the coil is $\frac{2\pi}{\beta}$ volt. The value of β is__.

Ans. (5)

Sol. $\phi = NAB\cos(\omega t)$

$$\varepsilon = -\frac{d\phi}{dt} = NAB\omega\sin(\omega t)$$

$\varepsilon_{max} = NAB\omega$

$= 200 \times 0.2 \times 0.01 \times \pi$

$$= \frac{4\pi}{10} = \frac{2\pi}{5}\text{volt}$$

58. In Young's double slit experiment, monochromatic light of wavelength 5000 Å is used. The slits are 1.0 mm apart and screen is placed at 1.0 m away from slits. The distance from the centre of the screen where intensity becomes half of the maximum intensity for the first time is___$\times 10^{-6}$ m.

Ans. (125)

Sol. Let intensity of light on screen due to each slit is I_0

So internity at centre of screen is $4I_0$

Intensity at distance y from centre-

$$I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi$$

$I_{max} = 4I_0$

$$\frac{I_{max}}{2} = 2I_0 = 2I_0 + 2I_0\cos\phi$$

$\cos\phi = 0$

$$\phi = \frac{\pi}{2}$$

$$K\Delta x = \frac{\pi}{2}$$

$$\frac{2\pi}{\lambda} d\sin\theta = \frac{\pi}{2}$$

$$\frac{2}{\lambda} d \times \frac{y}{D} = \frac{1}{2}$$

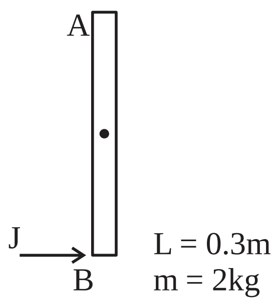
$$y = \frac{\lambda D}{4d} = \frac{5\times 10^{-7}\times 1}{4\times 10^{-3}}$$

$= 125 \times 10^{-6}$

$= 125$

59. A uniform rod AB of mass 2 kg and Length 30 cm at rest on a smooth horizontal surface. An impulse of force 0.2 Ns is applied to end B. The time taken by the rod to turn through at right angles will be $\frac{\pi}{x}$ s, where x = ____.

Ans. (4)

Sol. 

Impulse J = 0.2 N-S

$$J = \int F dt = 0.2 N - s$$

Angular impuls $(\vec{M})$

$$\vec{M}_c = \int \tau dt$$

$$= \int F \frac{L}{2} dt$$

$$= \frac{L}{2} \int F dt = \frac{L}{2} \times J$$

$$= \frac{0.3}{2} \times 0.2$$

= 0.03

$$I_{cm} = \frac{ML^2}{12} = \frac{2 \times (0.3)^2}{12} = \frac{0.09}{6}$$

$$M = I_{cm}(\omega_f - \omega_i)$$

$$0.03 = \frac{0.09}{6}(\omega_f)$$

$\omega_f = 2$ rad/s

$\theta = \omega t$

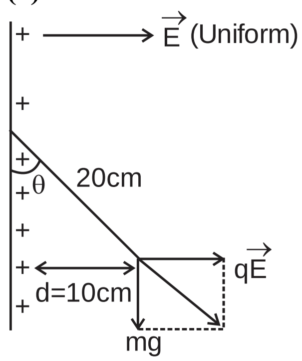
$$t = \frac{\theta}{\omega} = \frac{\pi}{2 \times 2} = \frac{\pi}{4} \text{sec}.$$

X = 4

60. Suppose a uniformly charged wall provides a uniform electric field of 2×10^4 N/C normally. A charged particle of mass 2 g being suspended through a silk thread of length 20 cm and remain stayed at a distance of 10 cm from the wall. Then the charge on the particle will be $\frac{1}{\sqrt{x}} \mu C$ where x =______. [use g = 10 m/s^2]

Ans. **(3)**

Sol.



$$\sin\theta = \frac{10}{20} = \frac{1}{2}$$

$\theta = 30°$

$$\tan\theta = \frac{qE}{mg}$$

$$\tan 30° = \frac{q \times 2 \times 10^4}{1 \times 10^{-3} \times 10}$$

$$\frac{1}{\sqrt{3}} = q \times 10^6$$

$$q = \frac{1}{\sqrt{3}} \times 10^{-6} C$$

x = 3

SECTION-A

61. The transition metal having highest 3^{rd} ionisation enthalpy is :

(1) Cr (2) Mn

(3) V (4) Fe

Ans. **(2)**

Sol. 3rd Ionisation energy : [NCERT Data]

V : 2833 KJ/mol

Cr : 2990 KJ/mol

Mn : 3260 KJ/mol

Fe : 2962 KJ/mol

alternative

Mn : $3d^5\ 4s^2$

Fe : $3d^6\ 4s^2$

Cr : $3d^5\ 4s^1$

V : $3d^3\ 4s^2$

So Mn has highest 3rd IE among all the given elements due to d^5 configuration.

62. Given below are two statements :

Statement (I) : A π bonding MO has lower electron density above and below the inter-nuclear asix.

Statement (II) : The π^* antibonding MO has a node between the nuclei.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are false

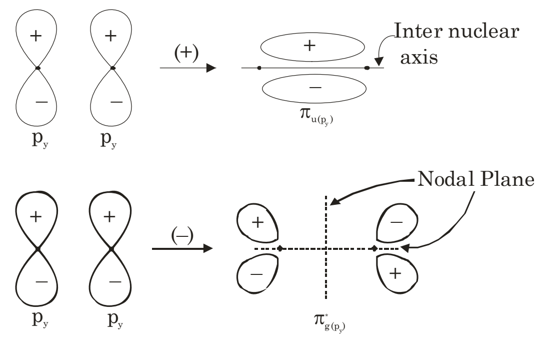
(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Ans. **(3)**

Sol. A π bonding molecular orbital has higher electron density above and below inter nuclear axis



63. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : In aqueous solutions Cr^{2+} is reducing while Mn^{3+} is oxidising in nature.

Reason (R) : Extra stability to half filled electronic configuration is observed than incompletely filled electronic configuration.

In the light of the above statement, choose the most appropriate answer from the options given below:

(1) Both (A) and (R) are true and (R) is the correct explanation of (A)

(2) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(3) (A) is false but (R) is true

(4) (A) is true but (R) is false

Ans. **(1)**

Sol. Cr^{2+} is reducing as it configuration changes from d^4 to d^3 due to formation of Cr^{3+}, which has half filled t_{2g} level, on other hand, the change Mn^{3+} to Mn^{2+} result half filled d^5 configuration which has extra stability.

64. Match List - I with List - II.

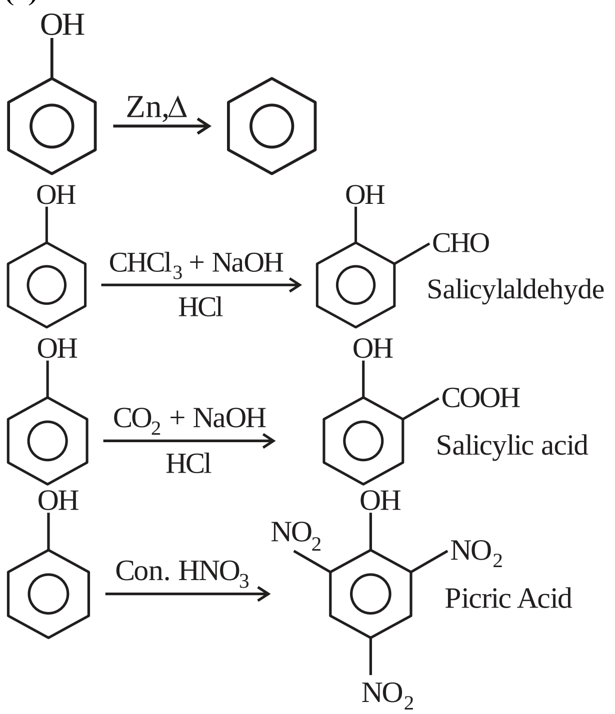
List-I (Reactants)	**List-II Products**
(A) Phenol, Zn/Δ	(I) Salicylaldehyde
(B) Phenol, $CHCl_3$, NaOH, HCl	(II) Salicylic acid
(C) Phenol, CO_2, NaOH, HCl	(III) Benzene
(D) Phenol, Conc. HNO_3	(IV) Picric acid

Choose the correct answer from the options given below.

(1) (A)-(IV), (B), (II), (C)-(I), (D)-(III)
(2) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(3) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Ans. **(3)**

Sol.



65. Given below are two statements :

Statement (I) : Both metal and non-metal exist in p and d-block elements.

Statement (II) : Non-metals have higher ionisation enthalpy and higher electronegativity than the metals.

In the light of the above statements, choose the most appropriate answer from the option given below:

(1) Both Statement I and Statement II are false
(2) Statement I is false but Statement II is true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are true

Ans. **(2)**

Sol. **I.** In p-Block both metals and non metals are present but in d-Block only metals are present.

II. EN and IE of non metals are greater than that of metals

I - False, **II-**True

66. The strongest reducing agent amont the following is:

(1) NH_3 (2) SbH_3
(3) BiH_3 (4) PH_3

Ans. **(3)**

Sol. Strongest reducing agent : BiH_3 explained by its low bond dissociation energy.

67. Which of the following compounds show colour due to d-d transition?

(1) $CuSO_4.5H_2O$ (2) $K_2Cr_2O_7$
(3) K_2CrO_4 (4) $KMnO_4$

Ans. **(1)**

Sol. $CuSO_4.5H_2O$

Cu^{2+} : $3d^9\,4s^0$

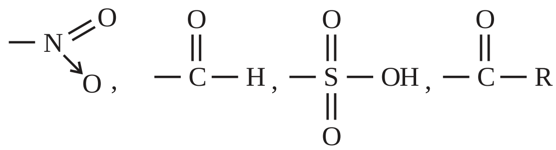
unpaired electron present so it show colour due to d-d transition.

68. The set of meta directing functional groups from the following sets is:

(1) –CN, $–NH_2$, –NHR, $–OCH_3$
(2) $–NO_2$, $–NH_2$, –COOH, –COOR
(3) $–NO_2$, –CHO, $–SO_3H$, –COR
(4) –CN, –CHO, $–NHCOCH_3$, –COOR

Ans. **(3)**

Sol.



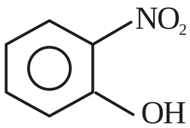
All are –M, Hence meta directing groups.

69. Select the compound from the following that will show intramolecular hydrogen bonding.

(1) H_2O

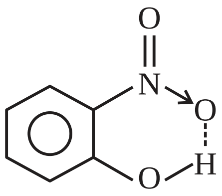
(2) NH_3

(3) C_2H_5OH

(4) 

Ans. (4)

Sol. H_2O, NH_3, C_2H_5OH ⇒ Intermolecular H-Bonding



⇒ Intramolecular H-Bonding

70. Lassaigne's test is used for detection of :

(1) Nitrogen and Sulphur only

(2) Nitrogen, Sulphur and Phosphorous Only

(3) Phosphorous and halogens only

(4) Nitrogen, Sulphur, phosphorous and halogens

Ans. (4)

Sol. Lassaigne's test is used for detection of all element N, S, P, X.

71. Which among the following has highest boiling point?

(1) $CH_3CH_2CH_2CH_3$

(2) $CH_3CH_2CH_2CH_2–OH$

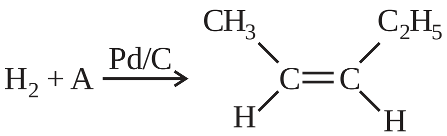
(3) $CH_3CH_2CH_2CHO$

(4) $H_5C_2 – O – C_2H_5$

Ans. (2)

Sol. Due to H-bonding boiling point of alcohol is High.

72. In the given reactions identify A and B.



$CH_3 - C \equiv C - CH_3 + H_2 \xrightarrow{Na/Liquid\,NH_3} "B"$

(1) A : 2–Pentyne B : trans – 2 – butene

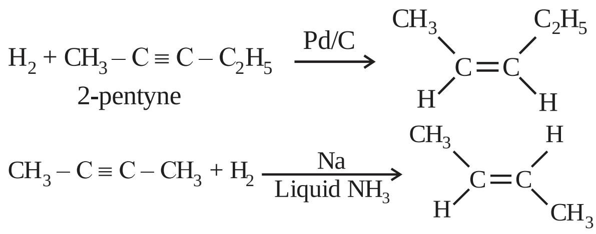
(2) A : n – Pentane B : trans – 2 – butene

(3) A : 2 – Pentyne B : Cis – 2 – butene

(4) A : n – Pentane B : Cis – 2 – butene

Ans. (1)

Sol.



Trans-2-butene

73. The number of radial node/s for 3p orbital is:

(1) 1 (2) 4

(3) 2 (4) 3

Ans. (1)

Sol. For 3p : n = 3, ℓ = 1

Number of radial node = n – ℓ – 1

= 3 – 1 – 1 = 1

74. Match List - I with List - II.

List - I	**List - II**
Compound	**Use**
(A) Carbon tetrachloride	(I) Paint remover
(B) Methylene chloride	(II) Refrigerators and air conditioners
(C) DDT	(III) Fire extinguisher
(D) Freons	(IV) Non Biodegradable insecticide

Choose the correct answer from the options given below :

(1) (A)-(I), (B), (II), (C)-(III), (D)-(IV)

(2) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(3) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Ans. (2)

Sol. CCl_4 used in fire extinguisher. CH_2Cl_2 used as paint remover. Freons used in refrigerator and AC. DDT used as non Biodegradable insecticide.

75. The functional group that shows negative resonance effect is:

(1) $–NH_2$ (2) $–OH$

(3) $–COOH$ (4) $–OR$

Ans. (3)

Sol. $-\overset{\overset{\displaystyle O}{\|}}{C}-OH$ shows –R effect, while rest 3 groups shows +R effect via lone pair.

76. $[Co(NH_3)_6]^{3+}$ and $[CoF_6]^{3-}$ are respectively known as:

(1) Spin free Complex, Spin paired Complex

(2) Spin paired Complex, Spin free Complex

(3) Outer orbital Complex, Inner orbital Complex

(4) Inner orbital Complex, Spin paired Complex

Ans. (2)

Sol. $[Co(NH_3)_6]^{3+}$

Co^{3+} (strong field ligand) $\Rightarrow 3d^6\left(t_{2g}^6, e_g^0\right)$,

Hybridisation : d^2sp^3

Inner obital complex(spin paired complex)

Pairing will take place.

$[CoF_6]^{3-}$

Co^{3+} (weak field ligand) $\Rightarrow 3d^6\left(t_{2g}^4, e_g^2\right)$

Hybridisation : sp^3d^2

Outer orbital complex (spin free complex)

no pairing will take place

77. Given below are two statements :

Statement (I) : SiO_2 and GeO_2 are acidic while SnO and PbO are amphoteric in nature.

Statement (II) : Allotropic forms of carbon are due to property of catenation and pπ-dπ bond formation.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are false

(2) Both Statement I and Statement II are true

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Ans. (3)

Sol. SiO_2 and GeO_2 are acidic and SnO, PbO are amphoteric.

Carbon does not have d-orbitals so can not form pπ-dπ Bond with itself. Due to properties of catenation and pπ-pπ bond formation. carbon is able to show allotropic forms.

78. $C_2H_5Br \xrightarrow{\text{alc. KOH}} A \xrightarrow[CCl_4]{Br_2} B \xrightarrow[\text{Excess}]{KCN} C \xrightarrow[\text{Excess}]{H_3O^+} D$

Acid D formed in above reaction is :

(1) Gluconic acid

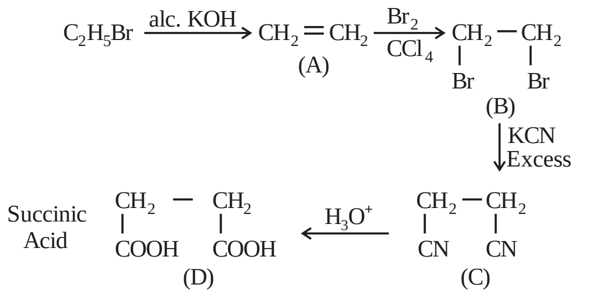
(2) Succinic acid

(3) Oxalic acid

(4) Malonic acid

Ans. (2)

Sol.



79. Solubility of calcium phosphate (molecular mass, M) in water is W_g per 100 mL at 25° C. Its solubility product at 25°C will be approximately.

(1) $10^7\left(\frac{W}{M}\right)^3$ (2) $10^7\left(\frac{W}{M}\right)^5$

(3) $10^3\left(\frac{W}{M}\right)^5$ (4) $10^5\left(\frac{W}{M}\right)^5$

Ans. (2)

Sol. $S = \frac{W \times 10}{M}$

$$Ca_3(PO_4)_2(s) \rightleftharpoons \underset{3s}{3Ca^{2+}(aq.)} + \underset{2s}{2PO_4^{3-}(aq.)}$$

$$S = \frac{W \times 1000}{M \times 100} = \frac{W \times 10}{M}$$

$K_{sp} = (3s)^3\,(2s)^2$

$= 108\,s^5$

$= 108 \times 10^5 \times \left(\frac{W}{M}\right)^5$

$= 1.08 \times 10^7\left(\frac{W}{M}\right)^5$

80. Given below are two statements :

Statement (I) : Dimethyl glyoxime forms a six-membered covalent chelate when treated with $NiCl_2$ solution in presence of NH_4OH.

Statement (II) : Prussian blue precipitate contains iron both in (+2) and (+3) oxidation states. In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is false but Statement II is true

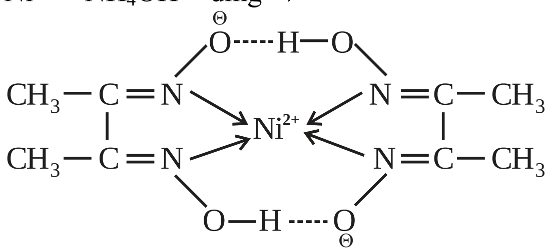
(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is true but Statement II is false

Ans. **(1)**

Sol. $Ni^{2+} + NH_4OH + dmg \rightarrow$



2 Five member ring

III II

$Fe_4[Fe(CN)_6]_3$

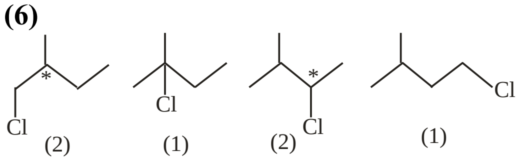
Prussian Blue

SECTION-B

81. Total number of isomeric compounds (including stereoisomers) formed by monochlorination of 2-methylbutane is________.

Ans. **(6)**

Sol.



(2) (1) (2) (1)

82. The following data were obtained during the first order thermal decomposition of a gas A at constant volume:

$A(g) \rightarrow 2B(g) + C(g)$

S.No	Time/s	Total pressure/(atm)
1.	0	0.1
2.	115	0.28

The rate constant of the reaction is______ $\times 10^{-2} s^{-1}$ (nearest integer)

Ans. **(2)**

Sol.

	A(g)	→	2B(g)	+	C(g)
t = 0	0.1				
t = 115 sec.	0.1 – x		2x		x

$0.1 + 2x = 0.28$

$2x = 0.18$

$x = 0.09$

$$K = \frac{1}{115} \ln \frac{0.1}{0.1 - 0.09}$$

$= 0.0200\ sec^{-1}$

$= 2 \times 10^{-2}\ sec^{-1}$

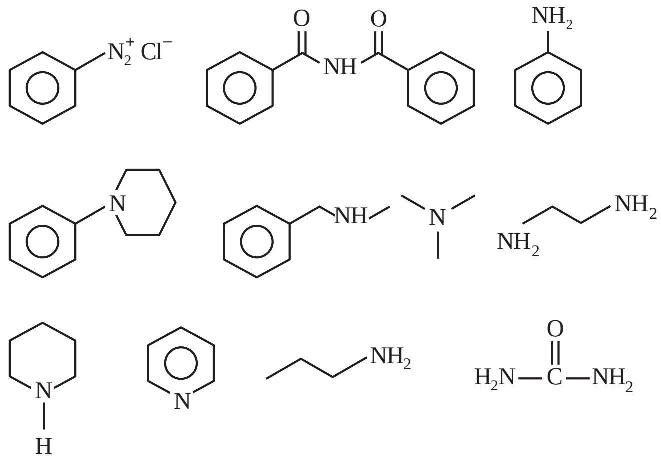
83. The number of tripeptides formed by three different amino acids using each amino acid once is ______.

Ans. **(6)**

Sol. Let 3 different amino acid are A, B, C then following combination of tripeptides can be formed-

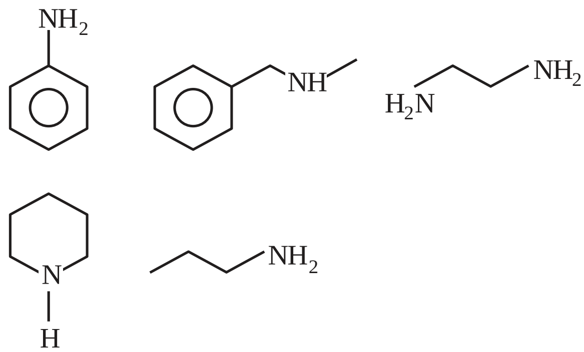
ABC, ACB, BAC, BCA, CAB, CBA

84. Number of compounds which give reaction with Hinsberg's reagent is_______.



Ans. **(5)**

Sol.



85. Mass of ethylene glycol (antifreeze) to be added to 18.6 kg of water to protect the freezing point at –24°C is________ kg (Molar mass in g mol^{-1} for ethylene glycol 62, K_f of water = 1.86 K kg mol^{-1})

Ans. (15)

Sol. $\Delta T_f = iK_f \times$ molality

$$24 = (1) \times 1.86 \times \frac{W}{62 \times 18.6}$$

W = 14880 gm

= 14.880 kg

86. Following Kjeldahl's method, 1g of organic compound released ammonia, that neutralised 10 mL of 2M H_2SO_4. The percentage of nitrogen in the compound is _______%.

Ans. (56)

Sol. $H_2SO_4 + 2NH_3 \rightarrow (NH_4)_2SO_4$

Millimole of $H_2SO_4 \rightarrow 10 \times 2$

So Millimole of $NH_3 = 20 \times 2 = 40$

Organic Compound $\rightarrow$ NH_3 40 Millimole

$\therefore$ Mole of N = $\frac{40}{1000}$

wt. of N = $\frac{40}{1000} \times 14$

% composition of N in organic compound

$= \frac{40 \times 14}{1000 \times 1} \times 100$

= 56%

87. The amount of electricity in Coulomb required for the oxidation of 1 mol of H_2O to O_2 is ____$\times 10^5$C.

Ans. (2)

Sol. $2H_2O \rightarrow O_2 + 4H^+ + 4e^-$

$$\frac{W}{E} = \frac{Q}{96500}$$

mole × n-factor = $\frac{Q}{96500}$

$1 \times 2 = \frac{Q}{96500}$

Q = 2 × 96500 C

= 1.93×10^5 C

88. For a certain reaction at 300K, K = 10, then ΔG° for the same reaction is –_______$\times 10^{-1}$ kJ mol^{-1}. (Given R = 8.314 JK^{-1} mol^{-1})

Ans. (57)

Sol. $\Delta G° = -RT \ell n K$

$= -8.314 \times 300 \ \ell n (10)$

= 5744.14 J/mole

= 57.44×10^{-1} kJ/mole

89. Consider the following redox reaction :

$$MnO_4^- + H^+ + H_2C_2O_4 \rightleftharpoons Mn^{2+} + H_2O + CO_2$$

The standard reduction potentials are given as below $\left(E^\circ_{red}\right)$

$$E^\circ_{MnO_4^-/Mn^{2+}} = +1.51V$$

$$E^\circ_{CO_2/H_2C_2O_4} = -0.49V$$

If the equilibrium constant of the above reaction is given as $K_{eq} = 10^x$, then the value of x = _______ (nearest integer)

Ans. (338 OR 339)

Sol. Cell Rx^n ; $MnO_4^- + H_2C_2O_4 \rightarrow Mn^{2+} + CO_2$

$E^\circ_{cell} = E^\circ_{op}$ of anode + E°_{RP} of cathode

= 0.49 + 1.51 = 2.00V

At equilibrium

$E_{cell} = 0$,

$$E^\circ_{cell} = \frac{0.059}{n}\log K$$

(As per NCERT $\frac{RT}{F} = 0.059$ But $\frac{RT}{F} = 0.0591$

can also be taken.)

$$2 = \frac{0.059}{10}\log K$$

logK = 338.98

90. 10 mL of gaseous hydrocarbon on combustion gives 40 mL of $CO_2(g)$ and 50 mL of water vapour. Total number of carbon and hydrogen atoms in the hydrocarbon is _____.

Ans. **(14)**

Sol. $\boxed{\begin{matrix} CxHy \\ 10ml \end{matrix}} + O_2 \rightarrow CO_2 + H_2O$

$$CxHy + \left(x + \frac{y}{4}\right)O_2 \rightarrow \underset{10x}{xCO_2} + \underset{5y}{\frac{y}{2}H_2O}$$

10x = 40

x = 4

5y = 50

y = 10

C_4H_{10}