FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Saturday 27th January, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. ${}^{n-1}C_r = (k^2-8)\,{}^nC_{r+1}$ if and only if :

(1) $2\sqrt{2} < k \le 3$ (2) $2\sqrt{3} < k \le 3\sqrt{2}$

(3) $2\sqrt{3} < k < 3\sqrt{3}$ (4) $2\sqrt{2} < k < 2\sqrt{3}$

Ans. (1)

Sol. ${}^{n-1}C_r = (k^2-8)\,{}^nC_{r+1}$

$$\underbrace{r+1\ge 0,\quad r\ge 0}_{r\ge 0}$$

$$\frac{{}^{n-1}C_r}{{}^nC_{r+1}} = k^2-8$$

$$\frac{r+1}{n} = k^2-8$$

$$\Rightarrow k^2-8>0$$

$$\left(k-2\sqrt{2}\right)\left(k+2\sqrt{2}\right)>0$$

$$k\in\left(-\infty,-2\sqrt{2}\right)\cup\left(2\sqrt{2},\infty\right) \qquad \ldots(\mathrm{I})$$

$$\therefore\ n\ge r+1,\ \frac{r+1}{n}\le 1$$

$$\Rightarrow k^2-8\le 1$$

$$k^2-9\le 0$$

$$-3\le k\le 3 \qquad \ldots(\mathrm{II})$$

From equation (I) and (II) we get

$$k\in\left[-3,-2\sqrt{2}\right)\cup\left(2\sqrt{2},3\right]$$

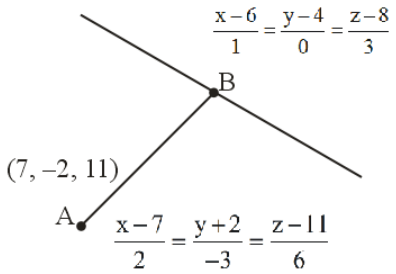
2. The distance, of the point (7, –2, 11) from the line $\frac{x-6}{1}=\frac{y-4}{0}=\frac{z-8}{3}$ along the line $\frac{x-5}{2}=\frac{y-1}{-3}=\frac{z-5}{6}$, is :

(1) 12 (2) 14

(3) 18 (4) 21

Ans. (2)

Sol. B = (2λ +7, –3λ – 2, 6λ + 11)



Point B lies on $\frac{x-6}{1}=\frac{y-4}{0}=\frac{z-8}{3}$

$$\frac{2\lambda+7-6}{1}=\frac{-3\lambda-2-4}{0}=\frac{6\lambda+11-8}{3}$$

$$-3\lambda-6=0$$

$$\lambda=-2$$

$$B\Rightarrow(3,4,-1)$$

$$AB=\sqrt{(7-3)^2+(4+2)^2+(11+1)^2}$$

$$=\sqrt{16+36+144}$$

$$=\sqrt{196}=14$$

3. Let x = x(t) and y = y(t) be solutions of the differential equations $\frac{dx}{dt}+ax=0$ and $\frac{dy}{dt}+by=0$ respectively, a, b ∈ R. Given that x(0) = 2; y(0) = 1 and 3y(1) = 2x(1), the value of t, for which x(t) = y(t), is :

(1) $\log_{\frac{2}{3}}2$ (2) $\log_4 3$

(3) $\log_3 4$ (4) $\log_{\frac{4}{3}}2$

Ans. (4)

Sol. $\frac{dx}{dt}+ax=0$

$\frac{dx}{x}=-adt$

$\int\frac{dx}{x}=-a\int dt$

$\ln|x|=-at+c$

at t = 0, x = 2

$\ln 2=0+c$

$\ln x=-at+\ln 2$

$\frac{x}{2}=e^{-at}$

$x=2e^{-at}$ ....(i)

$\frac{dy}{dt}+by=0$

$\frac{dy}{y}=-bdt$

$\ln|y|=-bt+\lambda$

t = 0, y = 1

$0 = 0 + \lambda$

$y = e^{-bt}$ ....(ii)

According to question

$3y(1) = 2x(1)$

$3e^{-b} = 2(2\ e^{-a})$

$e^{a-b}=\frac{4}{3}$

For x(t) = y(t)

$\Rightarrow\ 2e^{-at} = e^{-bt}$

$2 = e^{(a-b)t}$

$2=\left(\frac{4}{3}\right)^t$

$\log_{\frac{4}{3}}2=t$

4. If (a, b) be the orthocentre of the triangle whose vertices are (1, 2), (2, 3) and (3, 1), and $I_1=\int_a^b x\sin\left(4x-x^2\right)dx$, $I_2=\int_a^b \sin\left(4x-x^2\right)dx$, then $36\frac{I_1}{I_2}$ is equal to :

(1) 72 (2) 88

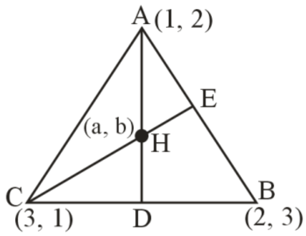
(3) 80 (4) 66

Ans. (1)

Sol. Equation of CE

$y - 1 = -(x - 3)$

$x + y = 4$



orthocentre lies on the line x + y = 4

so, a + b = 4

$I_1=\int_a^b x\sin\left(x(4-x)\right)dx$ ...(i)

Using king rule

$I_1=\int_a^b (4-x)\sin\left(x(4-x)\right)dx$ ...(ii)

(i) + (ii)

$2I_1=\int_a^b 4\sin\left(x(4-x)\right)dx$

$2I_1 = 4I_2$

$I_1 = 2I_2$

$\frac{I_1}{I_2}=2$

$\frac{36I_1}{I_2}=72$

5. If A denotes the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then :

(1) $A = B^3$ (2) $3A = B$

(3) $B = A^3$ (4) $A = 3B$

Ans. (1)

Sol. Sum of coefficients in the expansion of

$(1 - 3x + 10x^2)^n = A$

then $A = (1 - 3 + 10)^n = 8^n$ (put x = 1)

and sum of coefficients in the expansion of

$(1 + x^2)^n = B$

then $B = (1 + 1)^n = 2^n$

$A = B^3$

6. The number of common terms in the progressions 4, 9, 14, 19, ... , up to 25^{th} term and 3, 6, 9, 12, ..., up to 37^{th} term is :

(1) 9 (2) 5

(3) 7 (4) 8

Ans. (3)

Sol. 4, 9, 14, 19, …., up to 25^{th} term

$T_{25} = 4 + (25 - 1)\,5 = 4 + 120 = 124$

3, 6, 9, 12, …, up to 37^{th} term

$T_{37} = 3 + (37 - 1)3 = 3 + 108 = 111$

Common difference of I^{st} series $d_1 = 5$

Common difference of II^{nd} series $d_2 = 3$

First common term = 9, and

their common difference = 15 (LCM of d_1 and d_2)

then common terms are

9, 24, 39, 54, 69, 84, 99

7. If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to :

(1) 16 (2) 24

(3) 20 (4) 36

Ans. (3)

Sol. Equation of normal to parabola

$y = mx - 2m - m^3$

this normal passing through center of circle (2, 8)

$8 = 2m - 2m - m^3$

$m = -2$

So point P on parabola $\Rightarrow (am^2, -2am) = (4, 4)$

And C = (2, 8)

$PC = \sqrt{4+16} = \sqrt{20}$

$d^2 = 20$

8. If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and $\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is :

(1) 5 (2) 8

(3) 7 (4) 10

Ans. (2)

Sol. $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$

$\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$

the shortest distance between the lines

$$= \left|\frac{(\vec{a}-\vec{b})\cdot(\vec{d_1}\times\vec{d_2})}{|\vec{d_1}\times\vec{d_2}|}\right|$$

$$= \left|\frac{\begin{vmatrix}\lambda-4 & 0 & 2\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}}{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}}\right|$$

$$= \left|\frac{(\lambda-4)(-10+12)-0+2(4-4)}{|2\hat{i}-1\hat{j}+0\hat{k}|}\right|$$

$$\frac{6}{\sqrt{5}} = \left|\frac{2(\lambda-4)}{\sqrt{5}}\right|$$

$3 = |\lambda - 4|$

$\lambda - 4 = \pm 3$

$\lambda = 7, 1$

Sum of all possible values of λ is = 8

9. If $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}\,dx = a + b\sqrt{2} + c\sqrt{3}$, where a, b, c are rational numbers, then 2a + 3b – 4c is equal to :

(1) 4 (2) 10

(3) 7 (4) 8

Ans. (4)

Sol. $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}\,dx = \int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{(3+x)-(1+x)}\,dx$

$$\frac{1}{2}\left[\int_0^1 \sqrt{3+x}\,dx - \int_0^1 \left(\sqrt{1+x}\right)dx\right]$$

$$\frac{1}{2}\left[2\frac{(3+x)^{\frac{3}{2}}}{3}-\frac{2(1+x)^{\frac{3}{2}}}{3}\right]_0^1$$

$$\frac{1}{2}\left[\frac{2}{3}\left(8-3\sqrt{3}\right)-\frac{2}{3}\left(2^{\frac{3}{2}}-1\right)\right]$$

$$\frac{1}{3}\left[8-3\sqrt{3}-2\sqrt{2}+1\right]$$

$$=3-\sqrt{3}-\frac{2}{3}\sqrt{2}=a+b\sqrt{2}+c\sqrt{3}$$

$$a=3,\ b=-\frac{2}{3},\ c=-1$$

$2a + 3b - 4c = 6 - 2 + 4 = 8$

10. Let S = {l, 2, 3, ... , 10}. Suppose M is the set of all the subsets of S, then the relation

R = {(A, B): $A \cap B \neq \phi$; A, B $\in$ M} is :

(1) symmetric and reflexive only

(2) reflexive only

(3) symmetric and transitive only

(4) symmetric only

Ans. (4)

Sol. Let S = {1, 2, 3, …, 10}

R = {(A, B): $A \cap B \neq \phi$; A, B $\in$ M}

For Reflexive,

M is subset of 'S'

So $\phi \in$ M

for $\phi \cap \phi = \phi$

$\Rightarrow$ but relation is $A \cap B \neq \phi$

So it is not reflexive.

For symmetric,

ARB $\quad A \cap B \neq \phi$,

$\Rightarrow$ BRA $\quad \Rightarrow B \cap A \neq \phi$,

So it is symmetric.

For transitive,

If A = {(1, 2), (2, 3)}

B = {(2, 3), (3, 4)}

C = {(3, 4), (5, 6)}

ARB & BRC but A does not relate to C

So it not transitive

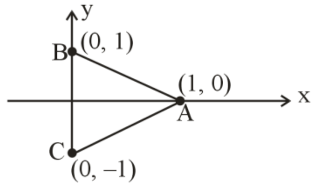
11. If S = {z $\in$ C : |z – i| = |z + i| = |z–1|}, then, n(S) is:

(1) 1 (2) 0

(3) 3 (4) 2

Ans. (1)

Sol. |z – i| = |z + i| = |z – 1|



ABC is a triangle. Hence its circum-centre will be the only point whose distance from A, B, C will be same.

So n(S) = 1

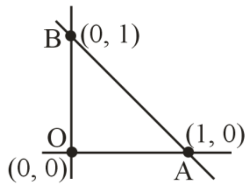
12. Four distinct points (2k, 3k), (1, 0), (0, 1) and (0, 0) lie on a circle for k equal to :

(1) $\frac{2}{13}$ (2) $\frac{3}{13}$

(3) $\frac{5}{13}$ (4) $\frac{1}{13}$

Ans. (3)

Sol. (2k, 3k) will lie on circle whose diameter is AB.



(x – 1) (x) + (y – 1) (y) = 0

$x^2 + y^2 - x - y = 0$ …(i)

Satisfy (2k, 3k) in (i)

$(2k)^2 + (3k)^2 - 2k - 3k = 0$

$13k^2 - 5k = 0$

$$k=0,\ k=\frac{5}{13}$$

hence $k=\frac{5}{13}$

13. Consider the function.

$$f(x)=\begin{cases}\dfrac{a\left(7x-12-x^2\right)}{b\left|x^2-7x+12\right|} & , \quad x<3 \\ 2^{\frac{\sin(x-3)}{x-[x]}} & , \quad x>3 \\ b & , \quad x=3\end{cases}$$

Where [x] denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that f(x) is continuous at x = 3, then the number of elements in S is :

(1) 2 (2) Infinitely many

(3) 4 (4) 1

Ans. (4)

Sol. $f\left(3^-\right)=\dfrac{a}{b}\dfrac{\left(7x-12-x^2\right)}{\left|x^2-7x+12\right|}$ (for f(x) to be cont.)

$\Rightarrow f(3^-) = \dfrac{-a}{b}\dfrac{(x-3)(x-4)}{(x-3)(x-4)}; x<3 \Rightarrow \dfrac{-a}{b}$

Hence $f\left(3^-\right)=\dfrac{-a}{b}$

Then $f\left(3^+\right)=2^{\lim\limits_{x\to3^+}\left(\frac{\sin(x-3)}{x-3}\right)}=2$ and

$f(3) = b$.

Hence $f(3) = f(3^+) = f(3^-)$

$\Rightarrow\ b = 2 = -\dfrac{a}{b}$

$b = 2, a = -4$

Hence only 1 ordered pair (–4, 2).

14. Let a_1, a_2, ….. a_{10} be 10 observations such that $\sum\limits_{k=1}^{10} a_k = 50$ and $\sum\limits_{\forall k<j} a_k \cdot a_j = 1100$. Then the standard deviation of $a_1, a_2, .., a_{10}$ is equal to :

(1) 5 (2) $\sqrt{5}$

(3) 10 (4) $\sqrt{115}$

Ans. (2)

Sol. $\sum\limits_{k=1}^{10} a_k = 50$

$a_1 + a_2 + \dots + a_{10} = 50$ ….(i)

$\sum\limits_{\forall k<j} a_k a_j = 1100$ ....(ii)

If $a_1 + a_2 + \dots + a_{10} = 50$.

$(a_1 + a_2 + \dots + a_{10})^2 = 2500$

$\Rightarrow \sum\limits_{i=1}^{10} a_i^2 + 2\sum\limits_{k<j} a_k a_j = 2500$

$\Rightarrow \sum\limits_{i=1}^{10} a_i^2 = 2500 - 2(1100)$

$\sum\limits_{i=1}^{10} a_i^2 = 300$, Standard deviation 'σ'

$= \sqrt{\dfrac{\sum a_i^2}{10} - \left(\dfrac{\sum a_i}{10}\right)^2} = \sqrt{\dfrac{300}{10} - \left(\dfrac{50}{10}\right)^2}$

$= \sqrt{30-25} = \sqrt{5}$

15. The length of the chord of the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$, whose mid point is $\left(1, \dfrac{2}{5}\right)$, is equal to :

(1) $\dfrac{\sqrt{1691}}{5}$ (2) $\dfrac{\sqrt{2009}}{5}$

(3) $\dfrac{\sqrt{1741}}{5}$ (4) $\dfrac{\sqrt{1541}}{5}$

Ans. (1)

Sol. Equation of chord with given middle point.

$T = S_1$

$\dfrac{x}{25}+\dfrac{y}{40}=\dfrac{1}{25}+\dfrac{1}{100}$

$\dfrac{8x+5y}{200}=\dfrac{8+2}{200}$

$y=\dfrac{10-8x}{5}$ …(i)

$$\frac{x^2}{25}+\frac{(10-8x)^2}{400}=1 \qquad \text{(put in original equation)}$$

$$\frac{16x^2+100+64x^2-160x}{400}=1$$

$4x^2 - 8x - 15 = 0$

$$x=\frac{8\pm\sqrt{304}}{8}$$

$$x_1=\frac{8+\sqrt{304}}{8};\ x_2=\frac{8-\sqrt{304}}{8}$$

$$\text{Similarly, } y=\frac{10-18\pm\sqrt{304}}{5}=\frac{2\pm\sqrt{304}}{5}$$

$$y_1=\frac{2-\sqrt{304}}{5};\ y_2=\frac{2+\sqrt{304}}{5}$$

$$\text{Distance} = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

$$=\sqrt{\frac{4\times304}{64}+\frac{4\times304}{25}}=\frac{\sqrt{1691}}{5}$$

16. The portion of the line 4x + 5y = 20 in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is :

(1) $\frac{8}{5}$ (2) $\frac{25}{41}$

(3) $\frac{2}{5}$ (4) $\frac{30}{41}$

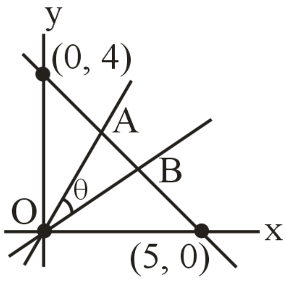
Ans. (4)

Sol. Co-ordinates of $A=\left(\frac{5}{3},\frac{8}{3}\right)$

Co-ordinates of $B=\left(\frac{10}{3},\frac{4}{3}\right)$

Slope of OA = $m_1=\frac{8}{5}$

Slope of OB = $m_2=\frac{2}{5}$



$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\tan\theta=\frac{\frac{6}{5}}{1+\frac{16}{25}}=\frac{30}{41}$$

$$\tan\theta=\frac{30}{41}$$

17. Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=3(\hat{i}-\hat{j}+\hat{k})$. Let $\vec{c}$ be the vector such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=3$. Then $\vec{a}\cdot\left(\left(\vec{c}\times\vec{b}\right)-\vec{b}-\vec{c}\right)$ is equal to :

(1) 32 (2) 24

(3) 20 (4) 36

Ans. (2)

Sol. $\vec{a}\cdot\left[\left(\vec{c}\times\vec{b}\right)-\vec{b}-\vec{c}\right]$

$\vec{a}\cdot\left(\vec{c}\times\vec{b}\right)-\vec{a}\cdot\vec{b}-\vec{a}\cdot\vec{c}$ ….(i)

given $\vec{a}\times\vec{c}=\vec{b}$

$\Rightarrow \left(\vec{a}\times\vec{c}\right)\cdot\vec{b}=\vec{b}\cdot\vec{b}=\left|\vec{b}\right|^2=27$

$\Rightarrow \vec{a}\cdot\left(\vec{c}\times\vec{b}\right)=\left[\vec{a}\ \vec{c}\ \vec{b}\right]=\left(\vec{a}\times\vec{c}\right)\cdot\vec{b}=27$ …(ii)

Now $\vec{a}\cdot\vec{b}=3-6+3=0$ …(iii)

$\vec{a}\cdot\vec{c}=3$ …(iv) (given)

By (i), (ii), (iii) & (iv)

27 – 0 – 3 = 24

18. If $a=\lim\limits_{x\to 0}\dfrac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$ and

$b=\lim\limits_{x\to 0}\dfrac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$, then the value of ab^3 is :

(1) 36 (2) 32 (3) 25 (4) 30

Ans. (2)

Sol. $a=\lim\limits_{x\to 0}\dfrac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$

$$=\lim_{x\to 0}\frac{\sqrt{1+x^4}-1}{x^4\left(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right)}$$

$$=\lim_{x\to 0}\frac{x^4}{x^4\left(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right)\left(\sqrt{1+x^4}+1\right)}$$

Applying limit $a=\dfrac{1}{4\sqrt{2}}$

$$b=\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$$

$$=\lim_{x\to 0}\frac{\left(1-\cos^2 x\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{2-\left(1+\cos x\right)}$$

$$b=\lim_{x\to 0}\left(1+\cos x\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

Applying limits $b=2\left(\sqrt{2}+\sqrt{2}\right)=4\sqrt{2}$

Now, $ab^3=\dfrac{1}{4\sqrt{2}}\times\left(4\sqrt{2}\right)^3=32$

19. Consider the matrix $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$.

Given below are two statements :

Statement I: f(–x) is the inverse of the matrix f(x).

Statement II: f(x) f(y) = f(x + y).

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Ans. (4)

Sol. $f(-x)=\begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

$$f(x)\cdot f(-x)=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

Hence statement- I is correct

Now, checking statement II

$$f(y)=\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(x)\cdot f(y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\Rightarrow f(x)\cdot f(y)=f(x+y)$

Hence statement-II is also correct.

20. The function $f : N - \{1\} \to N$; defined by f(n) = the highest prime factor of n, is :

(1) both one-one and onto

(2) one-one only

(3) onto only

(4) neither one-one nor onto

Ans. (4)

Sol. $f : N - \{1\} \to N$

f(n) = The highest prime factor of n.

f(2) = 2

f(4) = 2

$\Rightarrow$ many one

4 is not image of any element

$\Rightarrow$ into

Hence many one and into

Neither one-one nor onto.

SECTION-B

21. The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i}-2\hat{j}+2k$ and $\alpha\hat{i}+2\alpha\hat{j}-2k$ is acute, is _____.

Ans. (5)

Sol. $\cos\theta=\dfrac{\left(\alpha\hat{i}-2\hat{j}+2\hat{k}\right)\cdot\left(\alpha\hat{i}+2\alpha\hat{j}-2\hat{k}\right)}{\sqrt{\alpha^2+4+4}\ \sqrt{\alpha^2+4\alpha^2+4}}$

$\cos\theta=\dfrac{\alpha^2-4\alpha-4}{\sqrt{\alpha^2+8}\ \sqrt{5\alpha^2+4}}$

$\Rightarrow \alpha^2-4\alpha-4>0$

$\Rightarrow \alpha^2-4\alpha+4>8 \qquad \Rightarrow (\alpha-2)^2>8$

$\Rightarrow \alpha-2>2\sqrt{2}$ or $\alpha-2<-2\sqrt{2}$

$\alpha>2+2\sqrt{2}$ or $\alpha<2-2\sqrt{2}$

$\alpha\in(-\infty,-0.82)\cup(4.82,\infty)$

Least positive integral value of $\alpha \Rightarrow 5$

22. Let for a differentiable function $f:(0,\infty)\to R$, $f(x)-f(y)\ge \log_e\left(\dfrac{x}{y}\right)+x-y$, $\forall$ x, y $\in$ $(0,\infty)$. Then $\sum\limits_{n=1}^{20} f'\left(\dfrac{1}{n^2}\right)$ is equal to _____.

Ans. (2890)

Sol. $f(x)-f(y)\ge \ln x-\ln y+x-y$

$\dfrac{f(x)-f(y)}{x-y}\ge\dfrac{\ln x-\ln y}{x-y}+1$

Let $x>y$

$\lim\limits_{y\to x} f'\left(x^-\right)\ge\dfrac{1}{x}+1 \qquad \ldots..(1)$

Let $x<y$

$\lim\limits_{y\to x} f'\left(x^+\right)\le\dfrac{1}{x}+1 \qquad \ldots..(2)$

$f^1(x^-)=f^1(x^+)$

$f^1(x)=\dfrac{1}{x}+1$

$f'\left(\dfrac{1}{x^2}\right)=x^2+1$

$\sum\limits_{x=1}^{20}\left(x^2+1\right)=\sum\limits_{x-1}^{20}x^2+20$

$=\dfrac{20\times21\times41}{6}+20$

$=2890$

23. If the solution of the differential equation $(2x+3y-2)\,dx+(4x+6y-7)\,dy=0$, $y(0)=3$, is $\alpha x+\beta y+3\log_e|2x+3y-\gamma|=6$, then $\alpha+2\beta+3\gamma$ is equal to _____.

Ans. (29)

Sol. $2x+3y-2=t \qquad\qquad 4x+6y-4=2t$

$2+3\dfrac{dy}{dx}=\dfrac{dt}{dx} \qquad\qquad 4x+6y-7=2t-3$

$\dfrac{dy}{dx}=\dfrac{-(2x+3y-2)}{4x+6y-7}$

$\dfrac{dt}{dx}=\dfrac{-3t+4t-6}{2t-3}=\dfrac{t-6}{2t-3}$

$\int\dfrac{2t-3}{t-6}\,dt=\int dx$

$\int\left(\dfrac{2t-12}{t-6}+\dfrac{9}{t-6}\right)\cdot dt=x$

$2t+9\ln(t-6)=x+c$

$2(2x+3y-2)+9\ln(2x+3y-8)=x+c$

$x=0,\ y=3$

$c=14$

$4x+6y-4+9\ln(2x+3y-8)=x+14$

$x+2y+3\ln(2x+3y-8)=6$

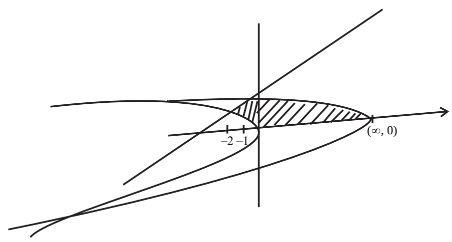
$\alpha=1,\ \beta=2,\ \gamma=8$

$\alpha+2\beta+3\gamma=1+4+24=29$

24. Let the area of the region $\{(x,y):\ x-2y+4\ge0,\ x+2y^2\ge0,\ x+4y^2\le8,\ y\ge0\}$ be $\dfrac{m}{n}$, where m and n are coprime numbers. Then m + n is equal to _____.

Ans. (119)

Sol.



$A=\int\limits_0^1\left[\left(8-4y^2\right)-\left(-2y^2\right)\right]dy+$

$\int\limits_1^{3/2}\left[\left(8-4y^2\right)-\left(2y-4\right)\right]dy$

$=\left[8y-\dfrac{2y^3}{3}\right]_0^1+\left[12y-y^2-\dfrac{4y^3}{3}\right]_1^{3/2}=\dfrac{107}{12}=\dfrac{m}{n}$

$\therefore\ m+n=119$

25. If

$$8=3+\frac{1}{4}(3+p)+\frac{1}{4^2}(3+2p)+\frac{1}{4^3}(3+3p)+....\infty,$$

then the value of p is _____.

Ans. (9)

Sol. $8=\frac{3}{1-\frac{1}{4}}+\frac{p\cdot\frac{1}{4}}{\left(1-\frac{1}{4}\right)^2}$

(sum of infinite terms of A.G.P = $\frac{a}{1-r}+\frac{dr}{(1-r)^2}$)

$\Rightarrow \frac{4p}{9}=4 \Rightarrow p=9$

26. A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let a = P(X = 3), b = P(X ≥ 3) and c = P(X ≥ 6 |X > 3). Then $\frac{b+c}{a}$ is equal to _____.

Ans. (12)

Sol. $a=P(X=3)=\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{25}{216}$

$b=P(X\geq 3)=\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\left(\frac{5}{6}\right)^3\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^4\cdot\frac{1}{6}+......$

$=\frac{\frac{25}{216}}{1-\frac{5}{6}}=\frac{25}{216}\times\frac{6}{1}=\frac{25}{36}$

$P(X\geq 6)=\left(\frac{5}{6}\right)^5\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^6\cdot\frac{1}{6}+.......$

$=\frac{\left(\frac{5}{6}\right)^5\cdot\frac{1}{6}}{1-\frac{5}{6}}=\left(\frac{5}{6}\right)^5$

$c=\frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3}=\frac{25}{36}$

$\frac{b+c}{a}=\frac{\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^2}{\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}}=12$

27. Let the set of all a ∈ R such that the equation $\cos 2x+a\sin x=2a-7$ has a solution be [p, q] and $r=\tan 9°-\tan 27°-\frac{1}{\cot 63°}+\tan 81°$, then pqr is equal to _____.

Ans. (48)

Sol. $\cos 2x + a\cdot\sin x = 2a - 7$

$a(\sin x-2)=2(\sin x-2)(\sin x+2)$

$\sin x=2,\quad a=2(\sin x+2)$

$\Rightarrow a \in [2, 6]$

$p=2\quad q=6$

$r = \tan 9° + \cot 9° - \tan 27 - \cot 27$

$r=\frac{1}{\sin 9\cdot\cos 9}-\frac{1}{\sin 27\cdot\cos 27}$

$=2\left[\frac{4}{\sqrt{5}-1}-\frac{4}{\sqrt{5}+1}\right]$

$r = 4$

$p\,.\,q\,.\,r = 2 \times 6 \times 4 = 48$

28. Let $f(x)=x^3+x^2f'(1)+xf''(2)+f'''(3)$, x ∈ R. Then $f'(10)$ is equal to _____.

Ans. (202)

Sol. $f(x) = x^3 + x^2\cdot f'(1) + x\cdot f''(2) + f'''(3)$

$f'(x)=3x^2+2xf'(1)+f''(2)$

$f''(x)=6x+2f'(1)$

$f'''(x)=6$

$f'(1) = -5,\ f''(2) = 2,\ f'''(3) = 6$

$f(x) = x^3 + x^2\cdot(-5) + x\cdot(2) + 6$

$f'(x) = 3x^2 - 10x + 2$

$f'(10) = 300 - 100 + 2 = 202$

29. Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $B = [B_1, B_2, B_3]$, where B_1, B_2, B_3 are column matrices, and $AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$,

$AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

If $\alpha = |B|$ and β is the sum of all the diagonal elements of B, then $\alpha^3 + \beta^3$ is equal to _____.

Ans. (28)

Sol. $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $\quad B = [B_1, B_2, B_3]$

$B_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$, $B_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$, $B_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$

$$AB_1 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 1, y_1 = -1, z_1 = -1$

$$AB_2 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$x_2 = 2, y_2 = 1, z_2 = -2$

$$AB_3 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$x_3 = 2, y_3 = 0, z_3 = -1$

$$B = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$\alpha = |B| = 3$

$\beta = 1$

$\alpha^3 + \beta^3 = 27 + 1 = 28$

30. If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, $A, B, C \geq 0$, then $5(3A - 2B - C)$ is equal to _____.

Ans. (5)

Sol. $x^2 + x + 1 = 0 \Rightarrow x = \omega, \omega^2 = \alpha$

Let $\alpha = \omega$

Now $(1 + \alpha)^7 = -\omega^{14} = -\omega^2 = 1 + \omega$

$A = 1, B = 1, C = 0$

$\therefore 5(3A - 2B - C) = 5(3 - 2 - 0) = 5$

SECTION-A

31. Position of an ant (S in metres) moving in Y-Z plane is given by $S = 2t^2\hat{j} + 5\hat{k}$ (where t is in second). The magnitude and direction of velocity of the ant at t = 1 s will be :

(1) 16 m/s in y-direction

(2) 4 m/s in x-direction

(3) 9 m/s in z-direction

(4) 4 m/s in y-direction

Ans. (4)

Sol. $\vec{v} = \frac{d\vec{s}}{dt} = 4t\hat{j}$

At t = 1 sec $\vec{v} = 4\hat{j}$

32. Given below are two statements :

Statement (I) : Viscosity of gases is greater than that of liquids.

Statement (II) : Surface tension of a liquid decreases due to the presence of insoluble impurities.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is correct but statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I and Statement II are incorrect

(4) Both Statement I and Statement II are correct

Ans. (2)

Sol. Gases have less viscosity.

Due to insoluble impurities like detergent surface tension decreases

33. If the refractive index of the material of a prism is $\cot\left(\frac{A}{2}\right)$, where A is the angle of prism then the angle of minimum deviation will be

(1) $\pi - 2A$

(2) $\frac{\pi}{2} - 2A$

(3) $\pi - A$

(4) $\frac{\pi}{2} - A$

Ans. (1)

Sol. $\cot\frac{A}{2} = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}$

$\Rightarrow \cos\frac{A}{2} = \sin\left(\frac{A+\delta_{min}}{2}\right)$

$\frac{A+\delta_{min}}{2} = \frac{\pi}{2} - \frac{A}{2}$

$\delta_{min} = \pi - 2A$

34. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If $\vec{E}$ and $\vec{B}$ represent the electric and magnetic fields respectively, then the region of space may have :

(A) $E = 0, B = 0$

(B) $E = 0, B \neq 0$

(C) $E \neq 0, B = 0$

(D) $E \neq 0, B \neq 0$

Choose the most appropriate answer from the options given below :

(1)(A), (B) and (C) only

(2) (A), (C) and (D) only

(3) (A), (B) and (D) only

(4) (B), (C) and (D) only

Ans. (3)

Sol. Net force on particle must be zero i.e. $q\vec{E} + q\vec{V} \times \vec{B} = 0$

Possible cases are

(i) $\vec{E}\ \&\ \vec{B} = 0$

(ii) $\vec{V} \times \vec{B} = 0, \vec{E} = 0$

(iii) $q\vec{E} = -q\vec{V} \times \vec{B}$

$\vec{E} \neq 0\ \&\ \vec{B} \neq 0$

35. The acceleration due to gravity on the surface of earth is g. If the diameter of earth reduces to half of its original value and mass remains constant, then acceleration due to gravity on the surface of earth would be :

(1) g/4 (2) 2g
(3) g/2 (4) 4g

Ans. (4)

Sol. $g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$

$\frac{g_2}{g_1} = \frac{R_1^2}{R_2^2}$

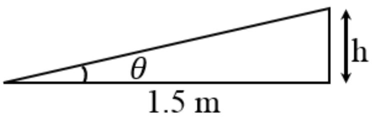
$g_2 = 4g_1 \left(R_2 = \frac{R_1}{2} \right)$

36. A train is moving with a speed of 12 m/s on rails which are 1.5 m apart. To negotiate a curve radius 400 m, the height by which the outer rail should be raised with respect to the inner rail is (Given, g = 10 m/s^2) :

(1) 6.0 cm (2) 5.4 cm
(3) 4.8 cm (4) 4.2 cm

Ans. (2)

Sol. $\tan\theta = \frac{v^2}{Rg} = \frac{12 \times 12}{10 \times 400}$

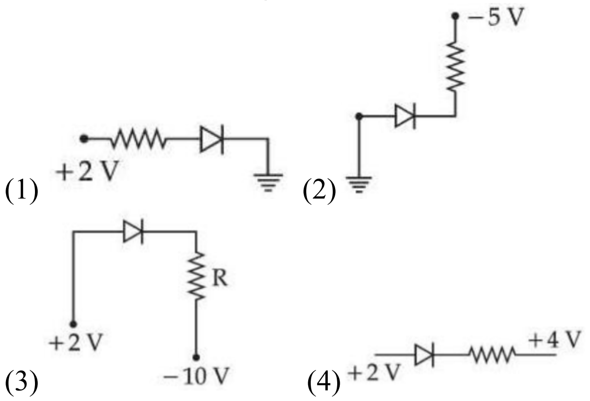


$\tan\theta = \frac{h}{1.5}$

$\Rightarrow \frac{h}{1.5} = \frac{144}{4000}$

h = 5.4 cm

37. Which of the following circuits is reverse - biased ?



Ans. (4)

Sol. P end should be at higher potential for forward biasing.

38. Identify the physical quantity that cannot be measured using spherometer :

(1) Radius of curvature of concave surface
(2) Specific rotation of liquids
(3) Thickness of thin plates
(4) Radius of curvature of convex surface

Ans. (2)

Sol. Spherometer can be used to measure curvature of surface.

39. Two bodies of mass 4 g and 25 g are moving with equal kinetic energies. The ratio of magnitude of their linear momentum is :

(1) 3 : 5 (2) 5 : 4
(3) 2 : 5 (4) 4 : 5

Ans. (3)

Sol. $\frac{P_1^2}{2m_1} = \frac{P_2^2}{2m_2}$

$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \frac{2}{5}$

40. 0.08 kg air is heated at constant volume through 5°C. The specific heat of air at constant volume is 0.17 kcal/kg°C and J = 4.18 joule/cal. The change in its internal energy is approximately.

(1) 318 J (2) 298 J
(3) 284 J (4) 142 J

Ans. (3)

Sol. Q = ΔU as work done is zero [constant volume]

$\Delta U = ms\,\Delta T$

= 0.08 × (170 × 4.18) × 5

≃ 284 J

41. The radius of third stationary orbit of electron for Bohr's atom is R. The radius of fourth stationary orbit will be:

(1) $\frac{4}{3}R$ (2) $\frac{16}{9}R$
(3) $\frac{3}{4}R$ (4) $\frac{9}{16}R$

Ans. (2)

Sol. $r \propto \frac{n^2}{Z}$

$\frac{r_4}{r_3} = \frac{4^2}{3^2}$

$r_4 = \frac{16}{9}R$

42. A rectangular loop of length 2.5 m and width 2 m is placed at 60° to a magnetic field of 4 T. The loop is removed from the field in 10 sec. The average emf induced in the loop during this time is

(1) – 2V (2) + 2V

(3) + 1V (4) – 1V

Ans. **(3)**

Sol. Average emf= $\frac{\text{Change in flux}}{\text{Time}} = -\frac{\Delta\phi}{\Delta t}$

$= -\frac{0-(4\times(2.5\times2)\cos60^\circ)}{10}$

$= +1V$

43. An electric charge $10^{-6}\mu C$ is placed at origin (0, 0) m of X –Y co-ordinate system. Two points P and Q are situated at $(\sqrt{3},\sqrt{3})$m and $(\sqrt{6},0)$m respectively. The potential difference between the points P and Q will be :

(1) $\sqrt{3}$V

(2) $\sqrt{6}$V

(3) 0 V

(4) 3 V

Ans. **(3)**

Sol. Potential difference $=\frac{KQ}{r_1}-\frac{KQ}{r_2}$

$r_1=\sqrt{(\sqrt{3})^2+(\sqrt{3})^2}$

$r_2=\sqrt{(\sqrt{6})^2+0}$

As $r_1=r_2=\sqrt{6}$m

So potential difference = 0

44. A convex lens of focal length 40 cm forms an image of an extended source of light on a photo-electric cell. A current I is produced. The lens is replaced by another convex lens having the same diameter but focal length 20 cm. The photoelectric current now is :

(1) $\frac{I}{2}$ (2) 4 I

(3) 2 I (4) I

Ans. **(4)**

Sol. As amount of energy incident on cell is same so current will remain same.

45. A body of mass 1000 kg is moving horizontally with a velocity 6 m/s. If 200 kg extra mass is added, the final velocity (in m/s) is:

(1) 6 (2) 2

(3) 3 (4) 5

Ans. **(4)**

Sol. Momentum will remain conserve

$1000 \times 6 = 1200 \times v$

v = 5 m/s

46. A plane electromagnetic wave propagating in x-direction is described by

$E_y = (200\ Vm^{-1})\sin[1.5 \times 10^7 t - 0.05\ x]$;

The intensity of the wave is :

(Use $\in_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$)

(1) 35.4 Wm^{-2} (2) 53.1 Wm^{-2}

(3) 26.6 Wm^{-2} (4) 106.2 Wm^{-2}

Ans. **(2)**

Sol. $I=\frac{1}{2}\varepsilon_0E_0^2\times c$

$I=\frac{1}{2}\times8.85\times10^{-12}\times4\times10^4\times3\times10^8$

$I = 53.1\ W/m^2$

47. Given below are two statements :

Statement (I) : Planck's constant and angular momentum have same dimensions.

Statement (II) : Linear momentum and moment of force have same dimensions.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Ans. **(1)**

Sol. $[h] = ML^2T^{-1}$

$[L] = ML^2T^{-1}$

$[P] = MLT^{-1}$

$[\tau] = ML^2T^{-2}$

(Here h is Planck's constant, L is angular momentum, P is linear momentum and τ is moment of force)

48. A wire of length 10 cm and radius $\sqrt{7}\times10^{-4}$m connected across the right gap of a meter bridge. When a resistance of 4.5 Ω is connected on the left gap by using a resistance box, the balance length is found to be at 60 cm from the left end. If the resistivity of the wire is R × 10^{-7}Ωm, then value of R is :

(1) 63 (2) 70

(3) 66 (4) 35

Ans. **(3)**

Sol. For null point,

$$\frac{4.5}{60}=\frac{R}{40}$$

Also, $R=\frac{\rho\ell}{A}=\frac{\rho\ell}{\pi r^2}$

$$4.5\times40=\rho\times\frac{0.1}{\pi\times7\times10^{-8}}\times60$$

$$\rho=66\times10^{-7}\Omega\times m$$

49. A wire of resistance R and length L is cut into 5 equal parts. If these parts are joined parallely, then resultant resistance will be :

(1) $\frac{1}{25}R$ (2) $\frac{1}{5}R$

(3) 25 R (4) 5 R

Ans. **(1)**

Sol. Resistance of each part = $\frac{R}{5}$

Total resistance = $\frac{1}{5}\times\frac{R}{5}=\frac{R}{25}$

50. The average kinetic energy of a monatomic molecule is 0.414 eV at temperature :

(Use K_B = 1.38 × 10^{-23} J/mol-K)

(1) 3000 K

(2) 3200 K

(3) 1600 K

(4) 1500 K

Ans. **(2)**

Sol. For monoatomic molecule degree of freedom = 3.

$\therefore\ K_{avg}=\frac{3}{2}K_BT$

$$T=\frac{0.414\times1.6\times10^{-19}\times2}{3\times1.38\times10^{-23}}$$

= 3200 K

SECTION-B

51. A particle starts from origin at t = 0 with a velocity $5\hat{i}$ m/s and moves in x-y plane under action of a force which produces a constant acceleration of $(3\hat{i}+2\hat{j})m/s^2$. If the x-coordinate of the particle at that instant is 84 m, then the speed of the particle at this time is $\sqrt{\alpha}$ m/s. The value of α is ______.

Ans. **(673)**

Sol u_x = 5 m/s a_x = 3 m/s^2 x = 84 m

$$v_x^2-u_x^2=2ax$$

$$v_x^2-25=2(3)(84)$$

V_x = 23 m/s

$$v_x-u_x=a_xt$$

$$t=\frac{23-5}{3}=6s$$

$v_y=0+a_yt$ = 0 + 2 × (6) = 12 m/s

$$v^2=v_x^2+v_y^2=23^2+12^2=673$$

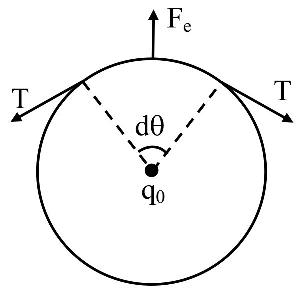
$$v=\sqrt{673}\ m/s$$

52. A thin metallic wire having cross sectional area of 10^{-4} m^2 is used to make a ring of radius 30 cm. A positive charge of 2π C is uniformly distributed over the ring, while another positive charge of 30 pC is kept at the centre of the ring. The tension in the ring is ______ N ; provided that the ring does not get deformed (neglect the influence of gravity).

(given, $\frac{1}{4\pi\epsilon_0}=9\times10^9$ SI units)

Ans. **(3)**

Sol.



$$2T\sin\frac{d\theta}{2}=\frac{kq_0}{R^2}\cdot\lambda Rd\theta$$

$$\left[\lambda=\frac{Q}{2\pi R}\right]$$

$$\Rightarrow T = \frac{Kq_0Q}{(R^2)\times 2\pi}$$

$$= \frac{(9\times10^9)(2\pi\times30\times10^{-12})}{(0.30)^2\times2\pi}$$

$$= \frac{9\times10^{-3}\times30}{9\times10^{-2}} = 3N$$

53. Two coils have mutual inductance 0.002 H. The current changes in the first coil according to the relation $i = i_0 \sin \omega t$, where $i_0 = 5A$ and $\omega = 50\pi$ rad/s. The maximum value of emf in the second coil is $\frac{\pi}{\alpha}$ V. The value of α is ____.

Ans. **(2)**

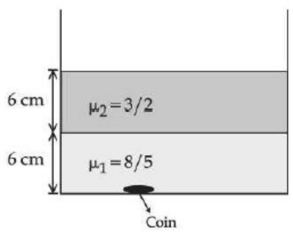
Sol. $\phi = Mi = Mi_0\sin\omega t$

$$EMF = -M\frac{di}{dt} = -0.002(i_0\omega\cos\omega t)$$

$$EMF_{max} = i_0\omega\ (0.002) = (5)(50\pi)(0.002)$$

$$EMF_{max} = \frac{\pi}{2}\ V$$

54. Two immiscible liquids of refractive indices $\frac{8}{5}$ and $\frac{3}{2}$ respectively are put in a beaker as shown in the figure. The height of each column is 6 cm. A coin is placed at the bottom of the beaker. For near normal vision, the apparent depth of the coin is $\frac{\alpha}{4}$ cm. The value of α is______.



Ans. **(31)**

Sol. $h_{app} = \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} = \frac{6}{3/2} + \frac{6}{8/5} = 4 + \frac{15}{4} = \frac{31}{4}$ cm

55. In a nuclear fission process, a high mass nuclide $(A \approx 236)$ with binding energy 7.6 MeV/Nucleon dissociated into middle mass nuclides $(A \approx 118)$, having binding energy of 8.6 MeV/Nucleon. The energy released in the process would be ____ MeV.

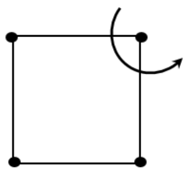
Ans. **(236)**

Sol. $Q = BE_{Product} - BE_{Rectant}$

$= 2(118)(8.6) - 236(7.6)$

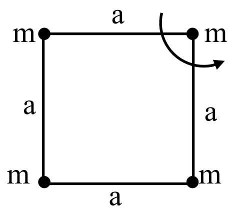
$= 236 \times 1 = 236$ MeV

56. Four particles each of mass 1 kg are placed at four corners of a square of side 2 m. Moment of inertia of system about an axis perpendicular to its plane and passing through one of its vertex is _____ kgm^2.



Ans. **(16)**

Sol.



$$I = ma^2 + ma^2 + m(\sqrt{2}a)^2$$

$= 4ma^2$

$= 4 \times 1 \times (2)^2 = 16$

57. A particle executes simple harmonic motion with an amplitude of 4 cm. At the mean position, velocity of the particle is 10 cm/s. The distance of the particle from the mean position when its speed becomes 5 cm/s is $\sqrt{\alpha}$ cm, where α = ______.

Ans. **(12)**

Sol. $V_{at\ mean\ position} = A\omega \Rightarrow 10 = 4\omega$

$$\omega = \frac{5}{2}$$

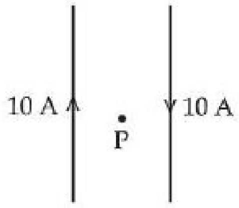
$$v = \omega\sqrt{A^2 - x^2}$$

$$5 = \frac{5}{2}\sqrt{4^2 - x^2} \Rightarrow x^2 = 16 - 4$$

$$x = \sqrt{12}\ cm$$

58. Two long, straight wires carry equal currents in opposite directions as shown in figure. The separation between the wires is 5.0 cm. The magnitude of the magnetic field at a point P midway between the wires is ____ μT

(Given : $\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$)



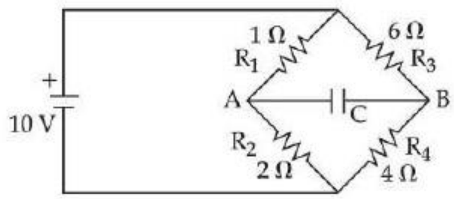
Ans. (160)

Sol. $B = \left(\frac{\mu_0 i}{2\pi a}\right) \times 2 = \frac{4\pi \times 10^{-7} \times 10}{\pi \times \left(\frac{5}{2} \times 10^{-2}\right)}$

$= 16 \times 10^{-5} = 160\mu T$

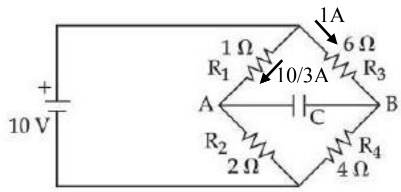
59. The charge accumulated on the capacitor connected in the following circuit is ____ μC

(Given C = 150 μF)



Ans. (400)

Sol.



$V_A + \frac{10}{3}(1) - 6(1) = V_B$

$V_A - V_B = 6 - \frac{10}{3} = \frac{8}{3}$ volt

$Q = C(V_A - V_B)$

$= 150 \times \frac{8}{3} = 400\mu C$

60. If average depth of an ocean is 4000 m and the bulk modulus of water is $2 \times 10^9\ Nm^{-2}$, then fractional compression $\frac{\Delta V}{V}$ of water at the bottom of ocean is $\alpha \times 10^{-2}$. The value of α is _____

(Given, $g = 10\ ms^{-2}$, $\rho = 1000\ kg\ m^{-3}$)

Ans. (2)

Sol. $B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$

$-\left(\frac{\Delta V}{V}\right) = \frac{\rho g h}{B} = \frac{1000 \times 10 \times 4000}{2 \times 10^9}$

$= 2 \times 10^{-2}$ [–ve sign represent compression]

SECTION-A

61. Two nucleotides are joined together by a linkage known as :

(1) Phosphodiester linkage

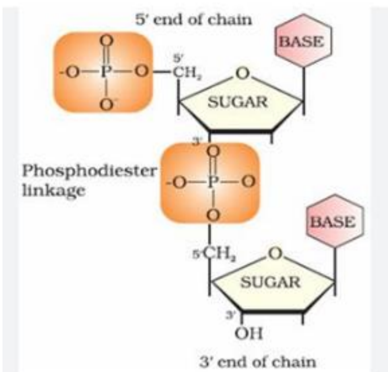
(2) Glycosidic linkage

(3) Disulphide linkage

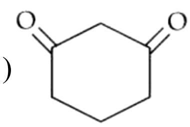
(4) Peptide linkage

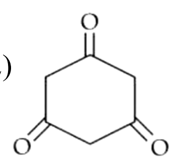
Ans. **(1)**

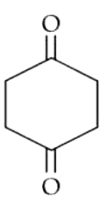
Sol. Phosphodiester linkage

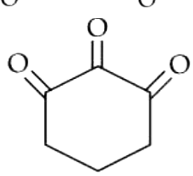


62. Highest enol content will be shown by :

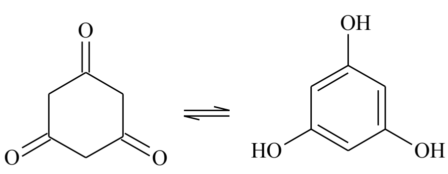
(1) 

(2) 

(3) 

(4) 

Ans. **(2)**

Sol. 

Aromatic

63. Element not showing variable oxidation state is :

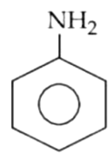
(1)Bromine (2)Iodine

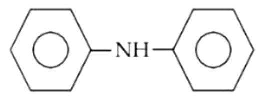
(3)Chlorine (4)Fluorine

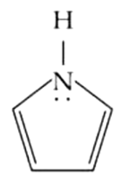
Ans. **(4)**

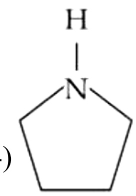
Sol. Fluorine does not show variable oxidation state.

64. Which of the following is strongest Bronsted base?

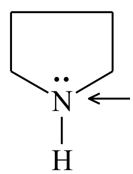
(1) 

(2) 

(3) 

(4) 

Ans. **(4)**

Sol.  localised lone pair& sp^3

65. Which of the following electronic configuration would be associated with the highest magnetic moment ?

(1) $[Ar]\ 3d^7$ (2) $[Ar]\ 3d^8$

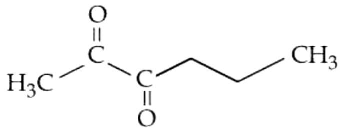
(3) $[Ar]\ 3d^3$ (4) $[Ar]\ 3d^6$

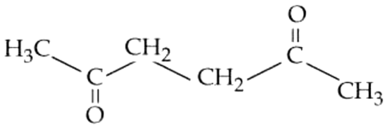
Ans. **(4)**

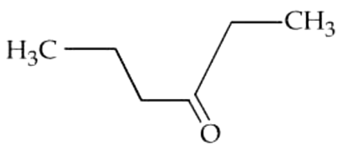
Sol.

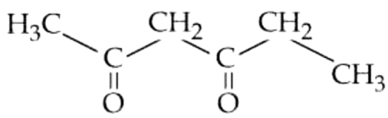
	$3d^7$	$3d^8$	$3d^3$	$3d^6$
No.of. unpaired e^-	3	2	3	4
Spin only Magnetic moment	$\sqrt{15}$ BM	$\sqrt{8}$ BM	$\sqrt{15}$ BM	$\sqrt{24}$ BM

66. Which of the following has highly acidic hydrogen?

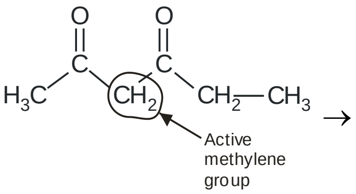
(1) 

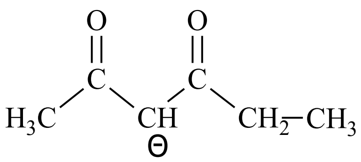
(2) 

(3) 

(4) 

Ans. (4)

Sol. 



Conjugate base is more stable due to more resonance of negative charge.

67. A solution of two miscible liquids showing negative deviation from Raoult's law will have :

(1) increased vapour pressure, increased boiling point

(2) increased vapour pressure, decreased boiling point

(3) decreased vapour pressure, decreased boiling point

(4) decreased vapour pressure, increased boiling point

Ans. (4)

Sol. Solution with negative deviation has

$P_T < P_A^0X_A + P_B^0X_B$

$P_A < P_A^0X_A$

$P_B < P_B^0X_B$

If vapour pressure decreases so boiling point increases.

68. Consider the following complex ions

$P = [FeF_6]^{3-}$

$Q = [V(H_2O)_6]^{2+}$

$R = [Fe(H_2O)_6]^{2+}$

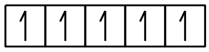
The correct order of the complex ions, according to their spin only magnetic moment values (in B.M.) is :

(1) R < Q < P (2) R < P < Q

(3) Q < R < P (4) Q < P < R

Ans. (3)

Sol. $[FeF_6]^{3-}$: Fe^{+3} : [Ar] $3d^5$

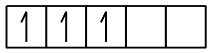
F : Weak field Ligand 

No. of unpaired electron's = 5

$\mu = \sqrt{5(5+2)}$

$\mu = \sqrt{35}$ BM

$[V(H_2O)_6]^{+2}$: V^{+2} : $3d^3$

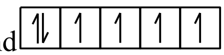


No. of unpaired electron's = 3

$\mu = \sqrt{3(3+2)}$

$\mu = \sqrt{15}$ BM

$[Fe(H_2O)_6]^{+2}$: Fe^{+2} : $3d^6$

H_2O : Weak field Ligand 

No. of unpaired electron's = 4

$\mu = \sqrt{4(4+2)}$

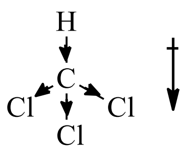
$\mu = \sqrt{24}$ BM

69. Choose the polar molecule from the following :

(1) CCl_4 (2) CO_2

(3) $CH_2 = CH_2$ (4) $CHCl_3$

Ans. (4)

Sol. 

$\mu \neq 0$

$CHCl_3$ is polar molecule and rest all molecules are non-polar.

70. Given below are two statements :

Statement (I) : The 4f and 5f - series of elements are placed separately in the Periodic table to preserve the principle of classification.

Statement (II) :S-block elements can be found in pure form in nature. In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Ans. **(3)**

Sol. s-block elements are highly reactive and found in combined state.

71. Given below are two statements :

Statement (I) : p-nitrophenol is more acidic than m-nitrophenol and o-nitrophenol.

Statement (II) : Ethanol will give immediate turbidity with Lucas reagent.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

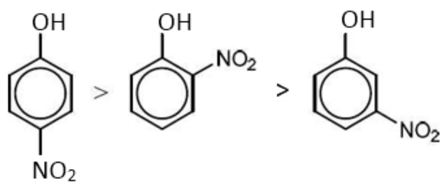
(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Ans. **(1)**

Sol. Acidic strength



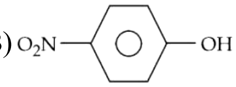
Ethanol give lucas test after long time

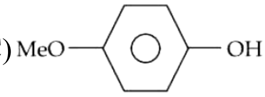
Statement (I)$\rightarrow$correct

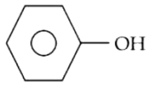
Statement (II) $\rightarrow$ incorrect

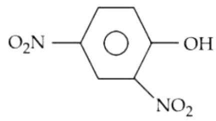
72. The ascending order of acidity of –OH group in the following compounds is :

(A) Bu – OH

(B) 

(C) 

(D) 

(E) 

Choose the correct answer from the options given below :

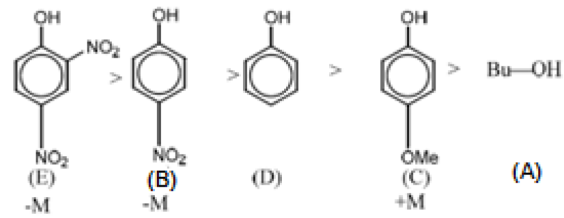
(1) (A) < (D) < (C) < (B) < (E)

(2) (C) < (A) < (D) < (B) < (E)

(3) (C) < (D) < (B) < (A) < (E)

(4) (A) < (C) < (D) < (B) < (E)

Ans. **(4)**

Sol. 

73. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Melting point of Boron (2453 K) is unusually high in group 13 elements.

Reason (R) : Solid Boron has very strong crystalline lattice.

In the light of the above statements, choose the most appropriate answer from the options given below ;

(1) Both (A) and (R) are correct but (R) Is not the correct explanation of (A)

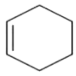
(2) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(3) (A) is true but (R) is false

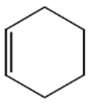
(4) (A) is false but (R) is true

Ans. **(2)**

Sol. Solid Boron has very strong crystalline lattice so its melting point unusually high in group 13 elements

74. Cyclohexene  is ________ type of an organic compound.
(1) Benzenoid aromatic
(2) Benzenoid non-aromatic
(3) Acyclic
(4) Alicyclic

Ans. **(4)**

Sol.  is Alicyclic

75. Yellow compound of lead chromate gets dissolved on treatment with hot NaOH solution. The product of lead formed is a :
(1) Tetraanionic complex with coordination number six
(2) Neutral complex with coordination number four
(3) Dianionic complex with coordination number six
(4) Dianionic complex with coordination number four

Ans. **(4)**

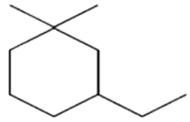
Sol. $PbCrO_4$ + NaOH (hot excess) $\rightarrow [Pb(OH)_4]^{-2}$ + Na_2CrO_4
Dianionic complex with coordination number four

76. Given below are two statements :
Statement (I) : Aqueous solution of ammonium carbonate is basic.
Statement (II) : Acidic/basic nature of salt solution of a salt of weak acid and weak base depends on K_a and K_b value of acid and the base forming it.
In the light of the above statements, choose the most appropriate answer from the options given below :
(1) Both Statement I and Statement II are correct
(2) Statement I is correct but Statement II is incorrect
(3) Both Statement I and Statement II are incorrect
(4) Statement I is incorrect but Statement II is correct

Ans. **(1)**

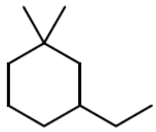
Sol. Aqueous solution of $(NH_4)_2CO_3$is Basic
pH of salt of weak acid and weak base depends on Ka and Kb value of acid and the base forming it

77. IUPAC name of following compound (P) is :


(P)

(1) l-Ethyl-5, 5-dimethylcyclohexane
(2) 3-Ethyl-1,1-dimethylcyclohexane
(3) l-Ethyl-3, 3-dimethylcyclohexane
(4) l,l-Dimethyl-3-ethylcyclohexane

Ans. **(2)**

Sol. 

3-ethy 1, 1 -dimethylcyclohexane

78. NaCl reacts with conc. H_2SO_4 and $K_2Cr_2O_7$ to give reddish fumes (B), which react with NaOH to give yellow solution (C). (B) and (C) respectively are ;
(1) CrO_2Cl_2, Na_2CrO_4 (2) Na_2CrO_4, CrO_2Cl_2
(3) CrO_2Cl_2, $KHSO_4$ (4) CrO_2Cl_2, $Na_2Cr_2O_7$

Ans. **(1)**

Sol. NaCl + conc. $H_2SO_4 + K_2Cr_2O_7$
$\rightarrow \underset{\text{(B)}}{CrO_2Cl_2} + KHSO_4 + NaHSO_4 + H_2O$
Reddish brown
$CrO_2Cl_2 + NaOH \rightarrow \underset{\text{(C)}}{Na_2CrO_4} + NaCl + H_2O$
Yellow colour

79. The correct statement regarding nucleophilic substitution reaction in a chiral alkyl halide is ;
(1) Retention occurs in S_Nl reaction and inversion occurs in S_N2 reaction.
(2) Racemisation occurs in S_Nl reaction and retention occurs in S_N2 reaction.
(3) Racemisation occurs in both S_N1 and S_N2 reactions.
(4) Racemisation occurs in S_N1 reaction and inversion occurs in S_N2 reaction.

Ans. **(4)**

Sol. SN^1 – Racemisation
SN^2 – Inversion

80. The electronic configuration for Neodymium is:

[Atomic Number for Neodymium 60]

(1)[Xe] $4f^4\ 6s^2$ (2) [Xe] $5f^4 7s^2$

(3) [Xe] $4f^6\ 6s^2$ (4) [Xe] $4f^1 5d^1 6s^2$

Ans. (1)

Sol. Electronic configuration of Nd(Z = 60) is;

[Xe] $4f^4\ 6s^2$

SECTION-B

81. The mass of silver (Molar mass of Ag : 108 $gmol^{-1}$) displaced by a quantity of electricity which displaces 5600 mL of O_2 at S.T.P. will be _____ g.

Ans. 107 gm or 108

Sol. Eq. of Ag = Eq. of O_2

Let x gm silver displaced,

$$\frac{x\times 1}{108}=\frac{5.6}{22.7}\times 4$$

(Molar volume of gas at STP = 22.7 lit)

x = 106.57 gm

Ans. 107

OR,

as per old STP data, molar volume=22.4 lit

$\frac{x\times 1}{108}=\frac{5.6}{22.4}\times 4$, x= 108 gm.

Ans. 108

82. Consider the following data for the given reaction

$2HI_{(g)} \rightarrow H_{2(g)} + I_{2(g)}$

	1	2	3
HI (mol L^{-1})	0.005	0.01	0.02
Rate (mol L^{-1}s–1)	7.5×10^{-4}	3.0×10^{-3}	1.2×10^{-2}

The order of the reaction is _________.

Ans. (2)

Sol. Let, R = $k[HI]^n$

using any two of given data,

$$\frac{3\times 10^{-3}}{7.5\times 10^{-4}}=\left(\frac{0.01}{0.005}\right)^n$$

n = 2

83. Mass of methane required to produce 22 g of CO_2 after complete combustion is ______g.

(Given Molar mass in g mol–1 C = 12.0

H = 1.0

O = 16.0)

Ans. (8)

Sol. $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

Moles of $CO_2 = \frac{22}{44} = 0.5$

So, required moles of CH_4 = 0.5

Mass = 0.5 × 16 = 8gm

84. If three moles of an ideal gas at 300 K expand isotherrnally from 30 dm^3 to 45 dm^3 against a constant opposing pressure of 80 kPa, then the amount of heat transferred is_______ J.

Ans. (1200)

Sol. Using, first law of thermodynamics,

$\Delta U = Q + W$,

$\Delta U = 0$: Process is isothermal

$Q = -W$

$W = -P_{ext}\Delta V$: Irreversible

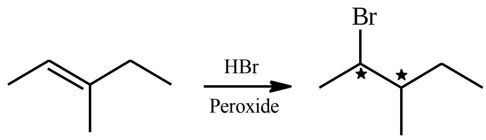
$= -80 \times 10^3\ (45 - 30) \times 10^{-3}$

$= -1200$ J

85. 3-Methylhex-2-ene on reaction with HBr in presence of peroxide forms an addition product (A). The number of possible stereoisomers for 'A' is_________.

Ans. (4)

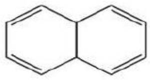
Sol.

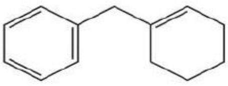


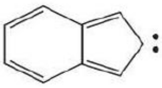
2chiral centres

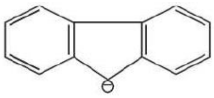
No. of stereoisomers = 4

86. Among the given organic compounds, the total number of aromatic compounds is

(A) 

(B) 

(C) 

(D) 

Ans. **(3)**

Sol. B,C and D are Aromatic

87. Among the following, total number of meta directing functional groups is (Integer based)

$-OCH_3$, $-NO_2$, $-CN$, $-CH_3$ $-NHCOCH_3$,

$-COR$, $-OH$, $-COOH$, $-Cl$

Ans. **(4)**

Sol. $-NO_2$, $-C\equiv N$, $-COR$, $-COOH$ are meta directing.

88. The number of electrons present in all the completely filled subshells having n=4 and $s=+\frac{1}{2}$ is ______.

(Where n = principal quantum number and s = spin quantum number)

Ans. **(16)**

Sol. $n = 4$ can have,

	4s	4p	4d	4f
Total e^-	2	6	10	14
Total e^- with $S=+\frac{1}{2}$	1	3	5	7

So, Ans.16

89. Sum of bond order of CO and NO^+ is _____.

Ans. **(6)**

Sol. $CO \Rightarrow \overset{-}{C}\equiv\overset{+}{O}$: BO = 3

$NO^+ \Rightarrow N\equiv O^+$: BO = 3

90. From the given list, the number of compounds with + 4 oxidation state of Sulphur ________.

SO_3, H_2SO_3, $SOCl_2$, SF_4, $BaSO_4$, $H_2S_2O_7$

Ans. **(3)**

Sol.

Compounds	SO_3	H_2SO_3	$SOCl_2$	SF_4	$BaSO_4$	$H_2S_2O_7$
O.S.of Sulphur:	+6	+4	+4	+4	+6	+6