FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Tuesday 30th January, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

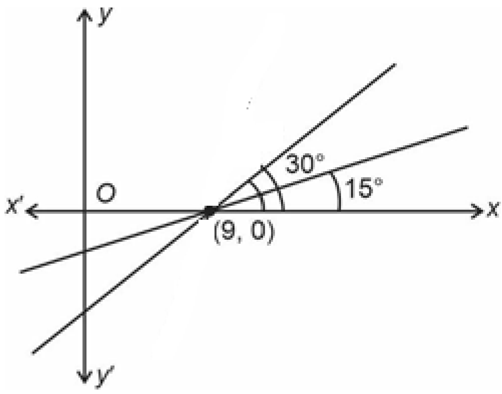
1. A line passing through the point A(9, 0) makes an angle of 30º with the positive direction of x-axis. If this line is rotated about A through an angle of 15º in the clockwise direction, then its equation in the new position is

(1) $\frac{y}{\sqrt{3}-2}+x=9$ (2) $\frac{x}{\sqrt{3}-2}+y=9$

(3) $\frac{x}{\sqrt{3}+2}+y=9$ (4) $\frac{y}{\sqrt{3}+2}+x=9$

Ans. (1)

Sol.



Eq^n : $y-0=\tan 15°\,(x-9) \Rightarrow y=(2-\sqrt{3})\,(x-9)$

2. Let S_a denote the sum of first n terms an arithmetic progression. If S_{20} = 790 and S_{10} = 145, then S_{15} – S_5 is :

(1) 395

(2) 390

(3) 405

(4) 410

Ans. (1)

Sol. $S_{20}=\frac{20}{2}[2a+19d]=790$

2a + 19d = 79 …..(1)

$S_{10}=\frac{10}{2}[2a+9d]=145$

2a + 9d = 29 …..(2)

From (1) and (2) a = -8, d = 5

$S_{15}-S_5=\frac{15}{2}[2a+14d]-\frac{5}{2}[2a+4d]$

$=\frac{15}{2}[-16+70]-\frac{5}{2}[-16+20]$

= 405-10

= 395

3. If z = x + iy, xy $\neq 0$, satisfies the equation $z^2+i\bar{z}=0$, then $|z^2|$ is equal to :

(1) 9

(2) 1

(3) 4

(4) $\frac{1}{4}$

Ans. (2)

Sol. $z^2=-i\bar{z}$

$|z^2|=|i\bar{z}|$

$|z^2| = |z|$

$|z|^2 - |z| = 0$

$|z|(|z| - 1) = 0$

$|z| = 0$ (not acceptable)

$\therefore\ |z| = 1$

$\therefore\ |z|^2 = 1$

4. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ be two vectors such that $|\vec{a}|=1$; $\vec{a}.\vec{b}=2$ and $|\vec{b}|=4$. If $\vec{c}=2(\vec{a}\times\vec{b})-3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to :

(1) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(2) $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(3) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(4) $\cos^{-1}\left(\frac{2}{3}\right)$

Ans. (3)

Sol. Given $|\vec{a}|=1, |\vec{b}|=4, \vec{a}.\vec{b}=2$

$\vec{c}=2(\vec{a}\times\vec{b})-3\vec{b}$

Dot product with $\vec{a}$ on both sides

$\vec{c}.\vec{a}=-6$ …..(1)

Dot product with $\vec{b}$ on both sides

$\vec{b}.\vec{c}=-48$ …..(2)

$\vec{c}.\vec{c}=4|\vec{a}\times\vec{b}|^2+9|\vec{b}|^2$

$|\vec{c}|^2=4\left[|a|^2|b|^2-(a.\vec{b})^2\right]+9|\vec{b}|^2$

$|\vec{c}|^2=4\left[(1)(4)^2-(4)\right]+9(16)$

$|\vec{c}|^2=4[12]+144$

$|\vec{c}|^2=48+144$

$|\vec{c}|^2=192$

$\therefore \cos\theta=\dfrac{\vec{b}.\vec{c}}{|\vec{b}||\vec{c}|}$

$\therefore \cos\theta=\dfrac{-48}{\sqrt{192}.4}$

$\therefore \cos\theta=\dfrac{-48}{8\sqrt{3}.4}$

$\therefore \cos\theta=\dfrac{-3}{2\sqrt{3}}$

$\therefore \cos\theta=\dfrac{-\sqrt{3}}{2} \Rightarrow \theta=\cos^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)$

5. The maximum area of a triangle whose one vertex is at (0, 0) and the other two vertices lie on the curve $y = -2x^2 + 54$ at points (x, y) and (-x, y) where y > 0 is :

(1) 88

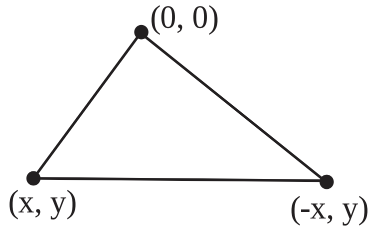
(2) 122

(3) 92

(4) 108

Ans. (4)

Sol.



Area of Δ

$=\dfrac{1}{2}\begin{vmatrix}0 & 0 & 1\\ x & y & 1\\ -x & y & 1\end{vmatrix}$

$\Rightarrow \left|\dfrac{1}{2}(xy+xy)\right|=|xy|$

$\text{Area}(\Delta)=|xy|=\left|x(-2x^2+54)\right|$

$\dfrac{d(\Delta)}{dx}=\left|(-6x^2+54)\right| \Rightarrow \dfrac{d\Delta}{dx}=0$ at x = 3

Area = 3 (-2 × 9 + 54) = 108

6. The value of $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{n^3}{(n^2+k^2)(n^2+3k^2)}$ is :

(1) $\dfrac{(2\sqrt{3}+3)\pi}{24}$

(2) $\dfrac{13\pi}{8(4\sqrt{3}+3)}$

(3) $\dfrac{13(2\sqrt{3}-3)\pi}{8}$

(4) $\dfrac{\pi}{8(2\sqrt{3}+3)}$

Ans. (2)

Sol. $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{n^3}{n^4\left(1+\dfrac{k^2}{n^2}\right)\left(1+\dfrac{3k^2}{n^2}\right)}$

$=\lim\limits_{n\to\infty}\dfrac{1}{n}\sum\limits_{k=1}^{n}\dfrac{n^3}{\left(1+\dfrac{k^2}{n^2}\right)\left(1+\dfrac{3k^2}{n^2}\right)}$

$=\displaystyle\int_0^1\dfrac{dx}{3(1+x^2)\left(\dfrac{1}{3}+x^2\right)}$

$$=\int_0^1 \frac{1}{3}\times\frac{3}{2}\frac{\left(x^2+1\right)-\left(x^2+\frac{1}{3}\right)}{\left(1+x^2\right)\left(x^2+\frac{1}{3}\right)}dx$$

$$=\frac{1}{2}\int_0^1\left[\frac{1}{x^2+\left(\frac{1}{\sqrt{3}}\right)^2}-\frac{1}{1+x^2}\right]dx$$

$$=\frac{1}{2}\left[\sqrt{3}\tan^{-1}\left(\sqrt{3}x\right)\right]_0^1-\frac{1}{2}\left(\tan^{-1}x\right)_0^1$$

$$=\frac{\sqrt{3}}{2}\left(\frac{\pi}{3}\right)-\frac{1}{2}\left(\frac{\pi}{4}\right)=\frac{\pi}{2\sqrt{3}}-\frac{\pi}{8}$$

$$=\frac{13\pi}{8.\left(4\sqrt{3}+3\right)}$$

7. Let $g : R\rightarrow R$ be a non constant twice differentiable such that $g'\left(\frac{1}{2}\right)=g'\left(\frac{3}{2}\right)$. If a real valued function f is defined as $f(x)=\frac{1}{2}\left[g(x)+g(2-x)\right]$, then

(1) $f''(x) = 0$ for atleast two x in (0, 2)

(2) $f''(x) = 0$ for exactly one x in (0, 1)

(3) $f''(x) = 0$ for no x in (0, 1)

(4) $f'\left(\frac{3}{2}\right)+f'\left(\frac{1}{2}\right)=1$

Ans. **(1)**

Sol. $f'(x)=\frac{g'(x)-g'(2-x)}{2}, f'\left(\frac{3}{2}\right)=\frac{g'\left(\frac{3}{2}\right)-g'\left(\frac{1}{2}\right)}{2}=0$

Also $f'\left(\frac{1}{2}\right)=\frac{g'\left(\frac{1}{2}\right)-g'\left(\frac{3}{2}\right)}{2}=0,\ f'\left(\frac{1}{2}\right)=0$

$\Rightarrow f'\left(\frac{3}{2}\right)=f'\left(\frac{1}{2}\right)=0$

$\Rightarrow \text{roots in}\left(\frac{1}{2},1\right)\text{and}\left(1,\frac{3}{2}\right)$

$\Rightarrow f''(x)$ is zero at least twice in $\left(\frac{1}{2},\frac{3}{2}\right)$

8. The area (in square units) of the region bounded by the parabola $y^2 = 4(x - 2)$ and the line $y = 2x - 8$

(1) 8

(2) 9

(3) 6

(4) 7

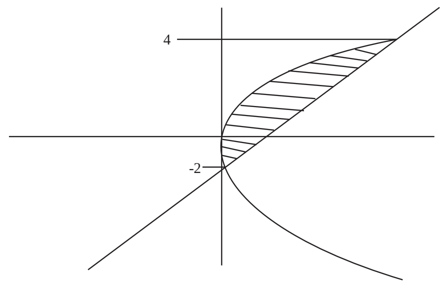
Ans. **(2)**

Sol. Let $X = x - 2$

$y^2 = 4x,\quad y = 2(x + 2) - 8$

$y^2 = 4x,\quad y = 2x - 4$

$$A=\int_{-2}^{4}\frac{y^2}{4}-\frac{y+4}{2}$$



$= 9$

9. Let $y = y(x)$ be the solution of the differential equation $\sec x\, dy + \{2(1 - x)\tan x + x(2 - x)\}\, dx = 0$ such that $y(0) = 2$. Then $y(2)$ is equal to :

(1) 2

(2) $2\{1 - \sin(2)\}$

(3) $2\{\sin(2) + 1\}$

(4) 1

Ans. **(1)**

Sol. $\frac{dy}{dx}=2(x-1)\sin x+\left(x^2-2x\right)\cos x$

Now both side integrate

$y(x)=\int 2(x-1)\sin x\,dx+\left[\left(x^2-2x\right)(\sin x)-\int(2x-2)\sin x\,dx\right]$

$y(x)=\left(x^2-2x\right)\sin x+\lambda$

$y(0)=0+\lambda\Rightarrow 2=\lambda$

$y(x)=\left(x^2-2x\right)\sin x+2$

$y(2)=2$

$\beta\ \gamma$ be the foot of perpendicular from the point (1, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$.

then $19(\alpha+\beta+\gamma)$ is equal to :

(1) 102

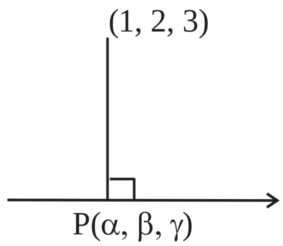
(2) 101

(3) 99

(4) 100

Ans. (2)

Sol.



Let foot P $(5k-3, 2k+1, 3k-4)$

$DR's \rightarrow AP: 5k-4,\ 2k-1,\ 3k-7$

$DR's \rightarrow Line: 5,\ 2,\ 3$

Condition of perpendicular lines $(25k-20)+(4k-2)+(9k-21)=0$

Then $k=\frac{43}{38}$

Then $19(\alpha+\beta+\gamma)$**=101**

11. Two integers x and y are chosen with replacement from the set {0, 1, 2, 3, ….., 10}. Then the probability that $|x-y|>5$ is :

(1) $\frac{30}{121}$

(2) $\frac{62}{121}$

(3) $\frac{60}{121}$

(4) $\frac{31}{121}$

Ans. (1)

Sol. If $x = 0, y = 6, 7, 8, 9, 10$

If $x = 1, y = 7, 8, 9, 10$

If $x = 2, y = 8, 9, 10$

If $x = 3, y = 9, 10$

If $x = 4, y = 10$

If $x = 5$, y = no possible value

Total possible ways = $(5 + 4 + 3 + 2 + 1) \times 2$

$= 30$

Required probability $=\frac{30}{11\times 11}=\frac{30}{121}$

12. If the domain of the function $f(x)=\cos^{-1}\left(\frac{2-|x|}{4}\right)+\left(\log_e(3-x)\right)^{-1}$ is $[-\alpha,\beta)-\{y\}$, then $\alpha+\beta+\gamma$ is equal to :

(1) 12

(2) 9

(3) 11

(4) 8

Ans. (3)

Sol. $-1\le\left|\frac{2-|x|}{4}\right|\le 1$

$\Rightarrow\left|\frac{2-|x|}{4}\right|\le 1$

$-4 \le 2 - |x| \le 4$

$-6 \le -|x| \le 2$

$-2 \le |x| \le 6$

$|x| \le 6$

$\Rightarrow\ x \in [-6, 6]$ …(1)

Now, $3 - x \ne 1$

And $x \ne 2$ …(2)

and $3 - x > 0$

$x < 3$ …(3)

From (1), (2) and (3)

$\Rightarrow\ x \in [-6, 3) - \{2\}$

$\alpha = 6$

$\beta = 3$

$\gamma = 2$

$\alpha + \beta + \gamma = 11$

13. Consider the system of linear equation $x + y + z = 4\mu$, $x + 2y + 2\lambda z = 10\mu$, $x + 3y + 4\lambda^2 z = \mu^2 + 15$, where $\lambda,\ \mu \in R$. Which one of the following statements is NOT correct ?

(1) The system has unique solution if $\lambda \ne \frac{1}{2}$ and $\mu \ne 1, 15$

(2) The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$

(3) The system has infinite number of solutions if $\lambda = \frac{1}{2}$ and $\mu = 15$

(4) The system is consistent if $\lambda \neq \frac{1}{2}$

Ans. (2)

Sol. $x + y + z = 4\mu$, $x + 2y + 2\lambda z = 10\mu$, $x + 3y + 4\lambda^2 z = \mu^2 + 15$,

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2\lambda \\ 1 & 3 & 4\lambda^2 \end{vmatrix} = (2\lambda - 1)^2$$

For unique solution $\Delta \neq 0,\ 2\lambda - 1 \neq 0,\ \left(\lambda \neq \frac{1}{2}\right)$

Let $\Delta = 0,\ \lambda = \frac{1}{2}$

$$\Delta_y = 0,\ \Delta_x = \Delta_z = \begin{vmatrix} 4\mu & 1 & 1 \\ 10\mu & 2 & 1 \\ \mu^2 + 15 & 3 & 1 \end{vmatrix}$$

$= (\mu - 15)(\mu - 1)$

For infinite solution $\lambda = \frac{1}{2},\ \mu = 1$ or 15

14. If the circles $(x+1)^2 + (y+2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ intersect at exactly two distinct points, then

(1) $5 < r < 9$

(2) $0 < r < 7$

(3) $3 < r < 7$

(4) $\frac{1}{2} < r < 7$

Ans. (3)

Sol. If two circles intersect at two distinct points

$\Rightarrow |r_1 - r_2| < C_1C_2 < r_1 + r_2$

$|r - 2| < \sqrt{9 + 16} < r + 2$

$|r - 2| < 5$ and $r + 2 > 5$

$-5 < r - 2 < 5$ $\qquad$ $r > 3$ ……….(2)

$-3 < r < 7$ $\qquad$ ………(1)

From (1) and (2)

$3 < r < 7$

15. If the length of the minor axis of ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is :

(1) $\frac{\sqrt{5}}{3}$

(2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{\sqrt{3}}$

(4) $\frac{2}{\sqrt{5}}$

Ans. (4)

Sol. $2b = ae$

$\frac{b}{a} = \frac{e}{2}$

$e = \sqrt{1 - \frac{e^2}{4}}$

$e = \frac{2}{\sqrt{5}}$

16. Let M denote the median of the following frequency distribution.

Class	0-4	4-8	8-12	12-16	16-20
Frequency	3	9	10	8	6

Then 20 M is equal to :

(1) 416

(2) 104

(3) 52

(4) 208

Ans. (4)

Sol.

Class	**Frequency**	**Cumulative frequency**
0-4	3	**3**
4-8	9	**12**
8-12	10	**22**
12-16	8	**30**
16-20	6	**36**

$$M = l + \left(\frac{\frac{N}{2} - C}{f}\right)h$$

$$M = 8 + \frac{18-12}{10} \times 4$$

M = 10.4

20M = 208

17. If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5} f'(0)$ is equal to ______

(1) 0

(2) 1

(3) 2

(4) 6

Ans. **(1)**

Sol. $\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^2 4x & \sin^2 2x \end{vmatrix}$

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3 & 0 & -3 \\ 0 & 3 & -3 \end{vmatrix}$

f(x) = 45

$f'(x) = 0$

18. Let A (2, 3, 5) and C(-3, 4, -2) be opposite vertices of a parallelogram ABCD if the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$ then the area of the parallelogram is equal to

(1) $\frac{1}{2}\sqrt{410}$

(2) $\frac{1}{2}\sqrt{474}$

(3) $\frac{1}{2}\sqrt{586}$

(4) $\frac{1}{2}\sqrt{306}$

Ans. **(2)**

Sol. Area = $\left|AC \times BD\right|$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 7 \\ 1 & 2 & 3 \end{vmatrix}$

$= \frac{1}{2}\left|-17\hat{i} - 8\hat{j} + 11\hat{k}\right| = \frac{1}{2}\sqrt{474}$

19. If $2\sin^3 x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, $n \in N$, then the roots of the equation $x^2 + nx + (n-3) = 0$ belong to :

(1) $(0, \infty)$

(2) $(-\infty, 0)$

(3) $\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$

(4) Z

Ans. **(2)**

Sol. $2\sin^3 x + 2\sin x.\cos^2 x + 4\sin x - 4 = 0$

$2\sin^3 x + 2\sin x.(1 - \sin^2 x) + 4\sin x - 4 = 0$

$6\sin x - 4 = 0$

$\sin x = \frac{2}{3}$

n = 5 (in the given interval)

$x^2 + 5x + 2 = 0$

$x = \frac{-5 \pm \sqrt{17}}{2}$

Required interval $(-\infty, 0)$

20. Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to R$ be a differentiable function such that $f(0) = \frac{1}{2}$, If the $\lim\limits_{x \to 0} \frac{x\int_0^x f(t)dt}{e^{x^2} - 1} = \alpha$, then $8\alpha^2$ is equal to :

(1) 16

(2) 2

(3) 1

(4) 4

Ans. **(2)**

Sol. $\lim\limits_{x\to 0}\dfrac{x\int_0^x f(t)\,dt}{\left(\dfrac{e^{x^2}-1}{x^2}\right)\times x^2}$

$\lim\limits_{x\to 0}\dfrac{\int_0^x f(t)\,dt}{x}$ $\quad\left(\lim\limits_{x\to 0}\dfrac{e^{x^2}-1}{x^2}=1\right)$

$=\lim\limits_{x\to 0}\dfrac{f(x)}{1}$ (using L Hospital)

$f(0)=\dfrac{1}{2}$

$\alpha=\dfrac{1}{2}$

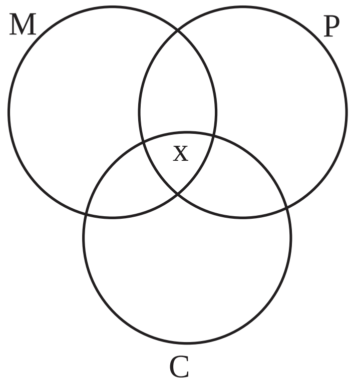
$8\alpha^2=2$

SECTION-B

21. A group of 40 students appeared in an examination of 3 subjects – Mathematics, Physics & Chemistry. It was found that all students passed in at least one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, at most 11 students passed in both Mathematics and Physics, at most 15 students passed in both Physics and Chemistry, at most 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is ________.

Ans. **(10)**

Sol.

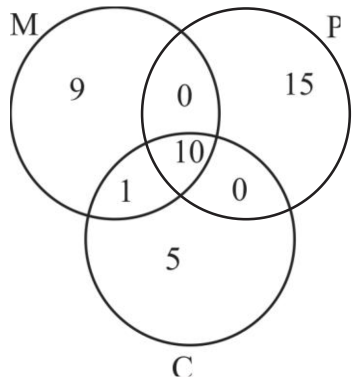


$11-x\geq 0$ (Maths and Physics)

$x\leq 11$

x =11 does not satisfy the data.

For x = 10



Hence maximum number of students passed in all the three subjects is 10.

22. If d_1 is the shortest distance between the lines $x+1=2y=-12z$, $x=y+2=6z-6$ and d_2 is the shortest distance between the lines $\dfrac{x-1}{2}=\dfrac{y+8}{-7}=\dfrac{z-4}{5}$, $\dfrac{x-1}{2}=\dfrac{y-2}{1}=\dfrac{z-6}{-3}$, then the value of $\dfrac{32\sqrt{3}d_1}{d_2}$ is :

Ans. **(16)**

Sol. $L_1:\dfrac{x+1}{1}=\dfrac{y}{1/2}=\dfrac{z}{-1/12}$, $L_2:\dfrac{x}{1}=\dfrac{y+2}{1}=\dfrac{z-1}{\frac{1}{6}}$

d_1 = shortest distance between L_1 & L_2

$=\left|\dfrac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{\left|(\vec{b}_1\times\vec{b}_2)\right|}\right|$

$d_1=2$

$L_3:\dfrac{x-1}{2}=\dfrac{y+8}{-7}=\dfrac{z-4}{5}$, $L_4:\dfrac{x-1}{2}=\dfrac{y-2}{1}=\dfrac{z-6}{-3}$

d_2 = shortest distance between L_3 & L_4

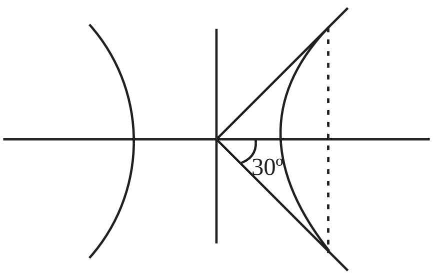
$d_2=\dfrac{12}{\sqrt{3}}$ Hence

$=\dfrac{32\sqrt{3}d_1}{d_2}=\dfrac{32\sqrt{3}\times 2}{\frac{12}{\sqrt{3}}}=16$

23. Let the latus rectum of the hyperbola $\frac{x^2}{9}-\frac{y^2}{b^2}=1$ subtend an angle of $\frac{\pi}{3}$ at the centre of the hyperbola. If b^2 is equal to $\frac{l}{m}\left(1+\sqrt{n}\right)$, where l and m are co-prime numbers, then $l^2 + m^2 + n^2$ is equal to __________

Ans. (182)

Sol. LR subtends 60° at centre



$$\Rightarrow \tan 30° = \frac{b^2/a}{ae} = \frac{b^2}{a^2e} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow e = \frac{\sqrt{3}b^2}{9}$$

Also, $e^2 = 1+\frac{b^2}{9} \Rightarrow 1+\frac{b^2}{9} = \frac{3b^4}{81}$

$$\Rightarrow b^4 = 3b^2 + 27$$

$$\Rightarrow b^4 - 3b^2 - 27 = 0$$

$$\Rightarrow b^2 = \frac{3}{2}(1+\sqrt{13})$$

$$\Rightarrow \ell = 3, m = 2, n = 13$$

$$\Rightarrow \ell^2 + m^2 + n^2 = 182$$

24. Let A = {1, 2, 3,….7} and let P(1) denote the power set of A. If the number of functions $f : A \to P(A)$ such that $a \in f(a), \forall a \in A$ is m^n, m and $n \in N$ and m is least, then m + n is equal to _______.

Ans. (44)

Sol. $f : A \to P(A)$

$a \in f(a)$

That means 'a' will connect with subset which contain element 'a'.

Total options for 1 will be 2^6. (Because 2^6 subsets contains 1)

Similarly, for every other element

Hence, total is $2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 = 2^{42}$

Ans. 2+42 = 44

25. The value $9\int_0^9 \left[\sqrt{\frac{10x}{x+1}}\right]dx$, where [t] denotes the greatest integer less than or equal to t, is _____.

Ans. (155)

Sol. $\frac{10x}{x+1} = 1 \quad \Rightarrow x = \frac{1}{9}$

$\frac{10x}{x+1} = 4 \quad \Rightarrow x = \frac{2}{3}$

$\frac{10x}{x+1} = 9 \quad \Rightarrow x = 9$

$$I = 9\left(\int_0^{1/9} 0dx + \int_{1/9}^{2/3} 1.dx + \int_{2/3}^{9} 2dx\right)$$

= 155

26. Number of integral terms in the expansion of $\left\{7^{\left(\frac{1}{2}\right)} + 11^{\left(\frac{1}{6}\right)}\right\}^{824}$ is equal to ________.

Ans. (138)

Sol. General term in expansion of $\left((7)^{1/2} + (11)^{1/6}\right)^{824}$ is

$$t_{r+1} = {}^{824}C_r\ (7)^{\frac{824-r}{2}}\ (11)^{r/6}$$

For integral term, r must be multiple of 6.

Hence r = 0, 6, 12, ……….822

27. Let y = y(x) be the solution of the differential equation $(1 - x^2)\,dy = \left[xy + (x^3+2)\sqrt{3(1-x^2)}\right]dx$, $-1 < x < 1, y(0) = 0$. If $y\left(\frac{1}{2}\right) = \frac{m}{n}$, m and n are co-prime numbers, then m + n is equal to ________.,

Ans. **(97)**

Sol. $\frac{dy}{dx} - \frac{xy}{1-x^2} = \frac{(x^3+2)\sqrt{3(1-x^2)}}{1-x^2}$

$IF = e^{-\int \frac{x}{1-x^2}dx} = e^{+\frac{1}{2}\ln(1-x^2)} = \sqrt{1-x^2}$

$y\sqrt{1-x^2} = \sqrt{3}\int (x^3+2)\,dx$

$y\sqrt{1-x^2} = \sqrt{3}\left(\frac{x^4}{4} + 2x\right) + c$

$\Rightarrow y(0) = 0 \qquad \therefore c = 0$

$y\left(\frac{1}{2}\right) = \frac{65}{32} = \frac{m}{n}$

$m + n = 97$

28. Let $\alpha, \beta \in N$ be roots of equation $x^2 - 70x + \lambda = 0$, where $\frac{\lambda}{2}, \frac{\lambda}{3} \notin N$. If λ assumes the minimum possible value, then $\frac{\left(\sqrt{\alpha-1}+\sqrt{\beta-1}\right)(\lambda+35)}{|\alpha-\beta|}$ is equal to :

Ans. **(60)**

Sol. $x^2 - 70x + \lambda = 0$

$\alpha + \beta = 70$

$\alpha\beta = \lambda$

$\therefore \alpha(70 - \alpha) = \lambda$

Since, 2 and 3 does not divide λ

$\therefore \alpha = 5, \beta = 65, \lambda = 325$

By putting value of α, β, λ we get the required value 60.

29. If the function $f(x) = \begin{cases} \frac{1}{|x|} & , |x| \ge 2 \\ ax^2 + 2b, & |x| < 2 \end{cases}$ is differentiable on R, then 48 (a + b) is equal to _____.

Ans. **(15)**

Sol. $f(x)\begin{cases} \frac{1}{x};\ x \ge 2 \\ ax^2 + 2b;\ -2 < x < 2 \\ -\frac{1}{x};\ x \le -2 \end{cases}$

Continuous at x = 2 $\Rightarrow \frac{1}{2} = \frac{a}{4} + 2b$

Continuous at x = -2 $\Rightarrow \frac{1}{2} = \frac{a}{4} + 2b$

Since, it is differentiable at x = 2

$-\frac{1}{x^2} = 2ax$

Differentiable at x = 2 $\Rightarrow \frac{-1}{4} = 4a \Rightarrow a = \frac{-1}{16}$, b $= \frac{3}{8}$

30. Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \ldots\ldots$ upto 10 terms and $\beta = \sum_{n=1}^{10} n^4$. If $4\alpha - \beta = 55k + 40$, then k is equal to ___________.

Ans. **(353)**

Sol. $\alpha = 1^2 + 4^2 + 8^2 \ldots.$

$t_n = an^2 + bn + c$

$$1 = a + b + c$$

$$4 = 4a + 2b + c$$

$$8 = 9a + 3b + c$$

On solving we get, $a = \frac{1}{2}$, $b = \frac{3}{2}$, $c = -1$

$$\alpha = \sum_{n=1}^{10}\left(\frac{n^2}{2} + \frac{3n}{2} - 1\right)^2$$

$$4\alpha = \sum_{n=1}^{10}\left(n^2 + 3n - 2\right)^2 \ , \ \beta = \sum_{n=1}^{10} n^4$$

$$4\alpha - \beta = \sum_{n=1}^{10}(6n^3 + 5n^2 - 12n + 4) = 55(353) + 40$$

SECTION-A

31. Match List-I with List-II.

	List-I		List-II
A.	Coefficient of viscosity	I.	$[M L^2T^{-2}]$
B.	Surface Tension	II.	$[M L^2T^{-1}]$
C.	Angular momentum	III.	$[M L^{-1}T^{-1}]$
D.	Rotational kinetic energy	IV.	$[M L^0T^{-2}]$

(1) A–II, B–I, C–IV, D–III
(2) A–I, B–II, C–III, D–IV
(3) A–III, B–IV, C–II, D–I
(4) A–IV, B–III, C–II, D–I

Ans. (3)

Sol. $F=\eta A\frac{dv}{dy}$

$\left[MLT^{-2}\right]=\eta\left[L^2\right]\left[T^{-1}\right]$

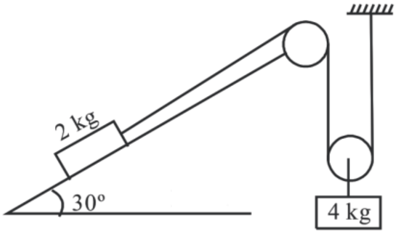
$\eta=\left[ML^{-1}T^{-1}\right]$

$S.T=\frac{F}{\ell}=\frac{\left[MLT^{-2}\right]}{[L]}=\left[ML^0T^{-2}\right]$

$L=mvr=\left[ML^2T^{-1}\right]$

$K.E=\frac{1}{2}I\omega^2=\left[ML^2T^{-2}\right]$

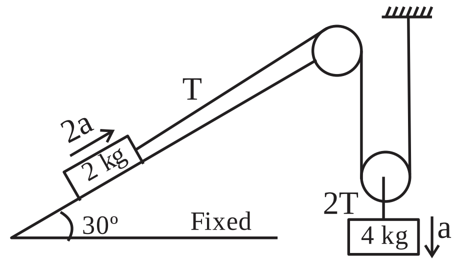
32. All surfaces shown in figure are assumed to be frictionless and the pulleys and the string are light. The acceleration of the block of mass 2 kg is :



(1) g
(2) $\frac{g}{3}$
(3) $\frac{g}{2}$
(4) $\frac{g}{4}$

Ans. (2)

Sol.

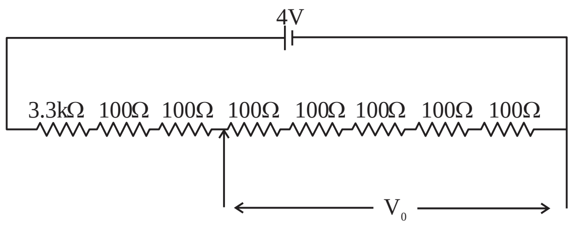


$40 - 2T = 4a$

$T - 10 = 4a \Rightarrow 20 = 12a$

$\Rightarrow a = 5/3 \Rightarrow 2a = \frac{g}{3}$

33. A potential divider circuit is shown in figure. The output voltage V_0 is



(1) 4V
(2) 2 mV
(3) 0.5 V
(4) 12 mV

Ans. (3)

Sol. $R_{eq}=4000\,\Omega$

$i=\frac{4}{4000}=\frac{1}{1000}A$

$V_0=i.R=\frac{1}{1000}\times 500=0.5V$

34. Young's modules of material of a wire of length 'L' and cross-sectional area A is Y. If the length of the wire is doubled and cross-sectional area is halved then Young's **modules** will be :

(1) $\frac{Y}{4}$
(2) $4Y$
(3) Y
(4) 2Y

Ans. (3)

Sol. Young's modulus depends on the material not length and cross sectional area. So young's modulus remains same.

35. The work function of a substance is 3.0 eV. The longest wavelength of light that can cause the emission of photoelectrons from this substance is approximately:

(1) 215 nm (2) 414 nm

(3) 400 nm (4) 200 nm

Ans. **(2)**

Sol. For P.E.E. : $\lambda \le \frac{hc}{W_e}$

$\lambda \le \frac{1240\, nm-eV}{3eV}$

$\lambda \le 413.33\, nm$

$\lambda_{\max} \approx 414\, nm$ for P.E.E.

36. The ratio of the magnitude of the kinetic energy to the potential energy of an electron in the 5th excited state of a hydrogen atom is :

(1) 4 (2) $\frac{1}{4}$

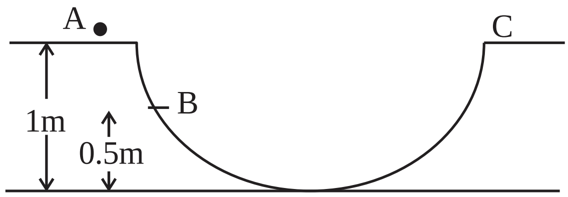
(3) $\frac{1}{2}$ (4) 1

Ans. **(3)**

Sol. $\frac{1}{2}|PE| = KE$ for each value of n (orbit)

$\therefore \frac{KE}{|PE|} = \frac{1}{2}$

37. A particle is placed at the point A of a frictionless track ABC as shown in figure. It is gently pushed toward right. The speed of the particle when it reaches the point B is : (Take g = 10 m/s^2).



(1) 20 m/s (2) $\sqrt{10}\, m/s$

(3) $2\sqrt{10}\, m/s$ (4) 10 m/s

Ans. **(2)**

Sol. By COME

$KE_A + U_A = KE_B + U_B$

$0 + mg(1) = \frac{1}{2}mv^2 + mg \times 0.5$

$v = \sqrt{g} = \sqrt{10}\, m/s$

38. The electric field of an electromagnetic wave in free space is represented as $\vec{E} = E_0 \cos(\omega t - kz)\hat{i}$. The corresponding magnetic induction vector will be :

(1) $\vec{B} = E_0 C \cos(\omega t - kz)\hat{j}$

(2) $\vec{B} = \frac{E_0}{C} \cos(\omega t - kz)\hat{j}$

(3) $\vec{B} = E_0 C \cos(\omega t + kz)\hat{j}$

(4) $\vec{B} = \frac{E_0}{C} \cos(\omega t + kz)\hat{j}$

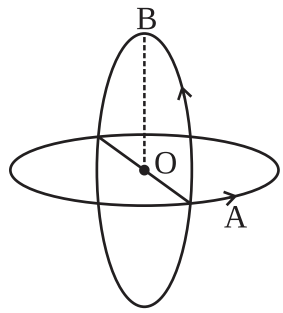
Ans. **(2)**

Sol. Given $\vec{E} = E_0 \cos(\omega t - kz)\hat{i}$

$\vec{B} = \frac{E_0}{C} \cos(\omega t - kz)\hat{j}$

$\hat{C} = \hat{E} \times \hat{B}$

39. Two insulated circular loop A and B radius 'a' carrying a current of 'I' in the anti clockwise direction as shown in figure. The magnitude of the magnetic induction at the centre will be :

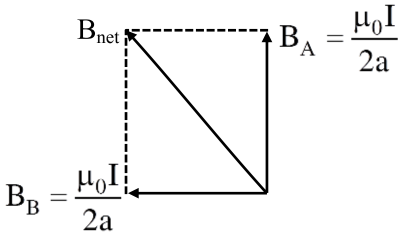


(1) $\frac{\sqrt{2}\mu_0 I}{a}$ (2) $\frac{\mu_0 I}{2a}$

(3) $\frac{\mu_0 I}{\sqrt{2}a}$ (4) $\frac{2\mu_0 I}{a}$

Ans. **(3)**

Sol.



$\because B_{net} = \frac{\sqrt{2}\mu_0 I}{2a}$

40. The diffraction pattern of a light of wavelength 400 nm diffracting from a slit of width 0.2 mm is focused on the focal plane of a convex lens of focal length 100 cm. The width of the 1st secondary maxima will be :

(1) 2 mm (2) 2 cm
(3) 0.02 mm (4) 0.2 mm

Ans. **(1)**

Sol. Width of 1st secondary maxima = $\frac{\lambda}{a}.D$

Here

$a = 0.2\times10^{-3} m$

$\lambda = 400\times10^{-9} m$

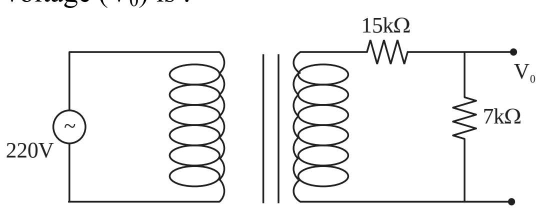
$D = 100\times10^{-2}$

Width of 1st secondary maxima

$= \frac{400\times10^{-9}}{0.2\times10^{-3}}\times100\times10^{-2}$

$= 2\ mm$

41. Primary coil of a transformer is connected to 220 V ac. Primary and secondary turns of the transforms are 100 and 10 respectively. Secondary coil of transformer is connected to two series resistance shown in shown in figure. The output voltage (V_0) is :



(1) 7 V (2) 15 V
(3) 44 V (4) 22 V

Ans. **(1)**

Sol. $\frac{\varepsilon_1}{\varepsilon_2} = \frac{N_1}{N_2} = \frac{100}{10} \Rightarrow \varepsilon_2 = 22V$

$I = \frac{22}{22\times10^3} = 1mA,\ V_0 = 7V$

42. The gravitational potential at a point above the surface of earth is $-5.12\times10^7\ J/kg$ and the acceleration due to gravity at that point is 6.4 m/s^2. Assume that the mean radius of earth to be 6400 km. The height of this point above the earth's surface is :

(1) 1600 km

(2) 540 km

(3) 1200 km

(4) 1000 km

Ans. **(1)**

Sol. $-\frac{GM_E}{R_E+h} = -5.12\times10^{-7}$ …. (i)

$\frac{GM_E}{(R_E+h)^2} = 6.4$ …. (ii)

By (i) and (ii)

$\Rightarrow h = 16\times10^5\ m = 1600\ km$

43. An electric toaster has resistance of 60 Ω at room temperature (27ºC). The toaster is connected to a 220 V supply. If the current flowing through it reaches 2.75 A, the temperature attained by toaster is around : (if $\alpha = 2\times10^{-4}/^oC$)

(1) 694ºC

(2) 1235ºC

(3) 1694ºC

(4) 1667ºC

Ans. **(3)**

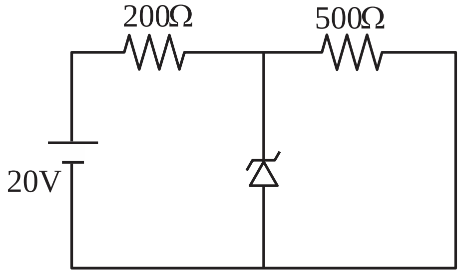
Sol. $R_{T=27} = 60\Omega,\ R_T = \frac{220}{2.75} = 80\Omega$

$R = R_0 (1+ \alpha\Delta T)$

$80 = 60 [1+ 2 \times 10^{-4}(T-27)]$

$T \approx 1694ºC$

44. A Zener diode of breakdown voltage 10V is used as a voltage regulator as shown in the figure. The current through the Zener diode is

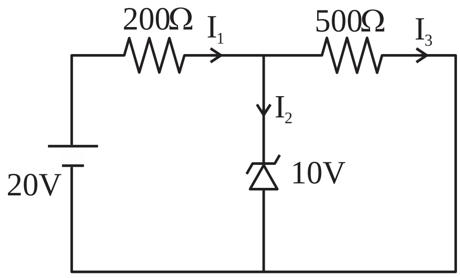


(1) 50 mA (2) 0

(3) 30 mA (4) 20 mA

Ans. (3)

Sol.



Zener is in breakdown region.

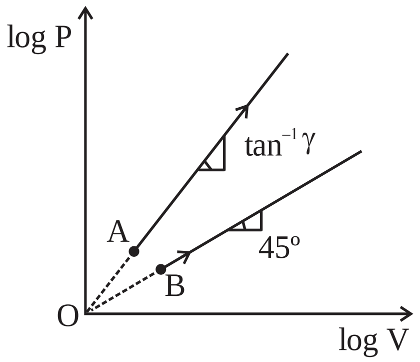
$$I_3 = \frac{10}{500} = \frac{1}{50}$$

$$I_1 = \frac{10}{200} = \frac{1}{20}$$

$$I_2 = I_1 - I_3$$

$$I_2 = \left(\frac{1}{20} - \frac{1}{50}\right) = \left(\frac{3}{100}\right) = 30\,mA$$

45. Two thermodynamical process are shown in the figure. The molar heat capacity for process A and B are C_A and C_B. The molar heat capacity at constant pressure and constant volume are represented by C_P and C_V, respectively. Choose the correct statement.



(1) $C_B = \infty, C_A = 0$

(2) $C_A = 0$ and $C_B = \infty$

(3) $C_P > C_V > C_A = C_B$

(4) $C_A > C_P > C_V$

Ans. (Bonus)

Sol. For process A

$$\log P = \gamma \log V \Rightarrow P = V^{\gamma}, (\gamma > 1)$$

$$PV^{-\gamma} = \text{Constant}$$

$$C_A = C_V + \frac{R}{1+\gamma} \quad \text{.... (i)}$$

Likewise for process B $\rightarrow$ $PV^{-1} = Constant$

$$C_B = C_v + \frac{R}{1+1}$$

$$C_B = C_v + \frac{R}{2} \quad \text{... (ii)}$$

$$C_P = C_v + R \quad \text{.... (iii)}$$

By (i), (ii) & (iii)

$C_P > C_B > C_A > C_v$ **[No answer matching]**

46. The electrostatic potential due to an electric dipole at a distance 'r' varies as :

(1) r (2) $\frac{1}{r^2}$

(3) $\frac{1}{r^3}$ (4) $\frac{1}{r}$

Ans. (2)

Sol. $V = \frac{kP\cos\theta}{r^2}$

& can also checked dimensionally

47. A spherical body of mass 100 g is dropped from a height of 10 m from the ground. After hitting the ground, the body rebounds to a height of 5m. The impulse of force imparted by the ground to the body is given by : (given g = 9.8 m/s^2)

(1) 4.32 kg ms^{-1} (2) 43.2 kg ms^{-1}

(3) 23.9 kg ms^{-1} (4) 2.39 kg ms^{-1}

Ans. (4)

Sol. $\vec{I} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$

M = 0.1 kg

$$I = \Delta P = 0.1\left(\sqrt{2\times9.8\times5} - \left(-\sqrt{2\times9.8\times10}\right)\right)$$

$$= 0.1\left(14 + 7\sqrt{2}\right) \approx 2.39 \text{ kg ms}^{-1}$$

48. A particle of mass m projected with a velocity 'u' making an angle of 30º with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is :

(1) $\frac{\sqrt{3}}{16}\frac{mu^3}{g}$ (2) $\frac{\sqrt{3}}{2}\frac{mu^2}{g}$

(3) $\frac{mu^3}{\sqrt{2}g}$ (4) zero

Ans. (1)

Sol. L = $mu\cos\theta H$

$$= mu\cos\theta \times \frac{u^2\sin^2\theta}{2g}$$

$$= \frac{mu^3}{2g}\times\frac{\sqrt{3}}{2}\times\left(\frac{1}{2}\right)^2 = \frac{\sqrt{3}mu^3}{16g}$$

49. At which temperature the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47ºC?

(1) 80 K (2) –73 K

(3) 4 K (4) 20 K

Ans. (4)

Sol. $\sqrt{\frac{3RT}{2}} = \sqrt{\frac{3R(320)}{32}}$

$$T = \frac{320}{16} = 20\,K$$

50. A series L,R circuit connected with an ac source E = (25 sin 1000 t) V has a power factor of $\frac{1}{\sqrt{2}}$. If the source of emf is changed to E = (20 sin 2000 t)V, the new power factor of the circuit will be :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{3}}$

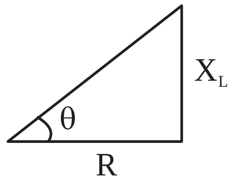
(3) $\frac{1}{\sqrt{5}}$ (4) $\frac{1}{\sqrt{7}}$

Ans. (3)

Sol. E = 25 sin (1000 t)

$$\cos\theta = \frac{1}{\sqrt{2}}$$

LR circuit



Initially $\frac{R}{\omega_1 L} = \frac{1}{\tan\theta} = \frac{1}{\tan 45^\circ} = 1$

$$X_L = \omega_1 L$$

$\omega_2 = 2\omega_1$, given

$$\tan\theta' = \frac{\omega_2 L}{R} = \frac{2\omega_1 L}{R}$$

$$tan\theta' = 2$$

$$\cos\theta' = \frac{1}{\sqrt{5}}$$

SECTION-B

51. The horizontal component of earth's magnetic field at a place is $3.5\times10^{-5}T$. A very long straight conductor carrying current of $\sqrt{2}A$ in the direction from South east to North West is placed. The force per unit length experienced by the conductor is ……… × 10^{-6} N/m.

Ans. (35)

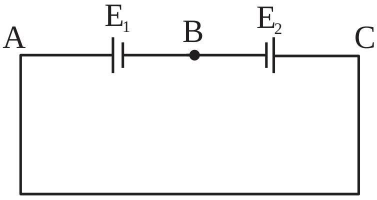
Sol. $B_H = 3.5\times10^{-5}T$

$F = i\ell B\sin\theta,\ \ \mathrm{i} = \sqrt{2}A$

$$\frac{F}{\ell} = iB\sin\theta = \sqrt{2}\times3.5\times10^{-5}\times\frac{1}{\sqrt{2}}$$

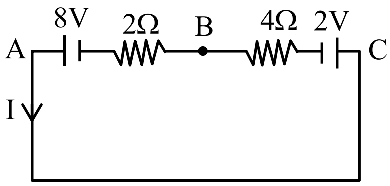
$$= 35\times10^{-6}\,N/m$$

52. Two cells are connected in opposition as shown. Cell E_1 is of 8 V emf and $2\,\Omega$ internal resistance; the cell E_2 is of 2 V emf and $4\,\Omega$ internal resistance. The terminal potential difference of cell E_2 is:



Ans. **(6)**

Sol.



$$I = \frac{8-2}{2+4} = \frac{6}{6} = 1A$$

Applying Kirchhoff from C to B

$$V_C - 2 - 4 \times 1 = V_B$$

$$V_C - V_B = 6V$$

= 6V

53. A electron of hydrogen atom on an excited state is having energy E_n = – 0.85 eV. The maximum number of allowed transitions to lower energy level is ….. . .

Ans. **(6)**

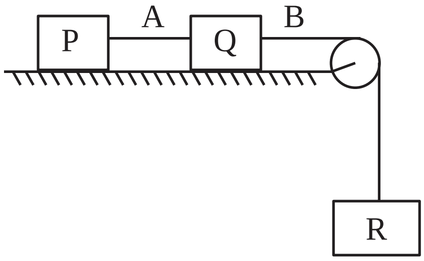
Sol. $E_n = -\frac{13.6}{n^2} = -0.85$

$\Rightarrow n = 4$

No of transition

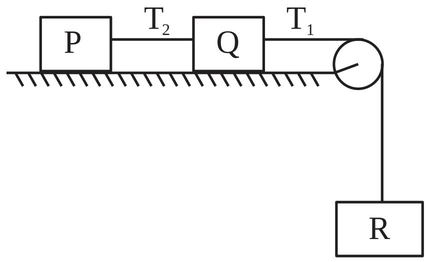
$$= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

54. Each of three blocks P, Q and R shown in figure has a mass of 3 kg. Each of the wire A and B has cross-sectional area 0.005 cm^2 and Young's modulus 2 × 10^{11} N m^{-2}. Neglecting friction, the longitudinal strain on wire B is _____ × 10^{-4}. (Take g = 10 m/s^2)



Ans. **(2)**

Sol.



$$a = \frac{10}{3} m/s^2$$

$$30 - T_1 = 3 \times a$$

$$T_1 = 20\,N$$

$$strain = \frac{stress}{Y}$$

$$= 2 \times 10^{-4}$$

55. The distance between object and its two times magnified real image as produced by a convex lens is 45 cm. The focal length of the lens used is ______ cm.

Ans. **(10)**

Sol. $\frac{v}{u} = -2$

$v = -2u$ **…(i)**

$v - u = 45$ ...(*ii*)

$\Rightarrow u = -15cm$

$v = 30cm$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$f = +10\ cm$$

56. The displacement and the increase in the velocity of a moving particle in the time interval of t to (t + 1) s are 125 m and 50 m/s, respectively. The distance travelled by the particle in (t + 2)th s is ____ m.

Ans. (175)

Sol. Considering acceleration is constant

$$v = u + at$$
$$u + 50 = u + a \Rightarrow a = 50\ m/s^2$$
$$125 = ut + \frac{1}{2}at^2$$
$$125 = u + \frac{a}{2}$$
$$\Rightarrow u = 100\ \ m/s$$

$$\therefore S_{n^{th}} = u + \frac{a}{2}[2n - 1]$$

= 175 m

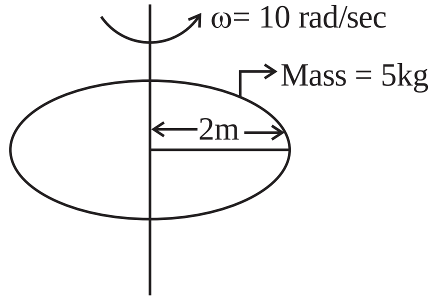
57. A capacitor of capacitance C and potential V has energy E. It is connected to another capacitor of capacitance 2 C and potential 2V. Then the loss of energy is $\frac{x}{3}E,$ where x is _____ .

Ans. (2)

Sol. Energy loss = $\frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

$$= \frac{2}{3}.E$$
$$\therefore x = 2$$

58. Consider a Disc of mass 5 kg, radius 2m, rotating with angular velocity of 10 rad/s about an axis perpendicular to the plane of rotation. An identical disc is kept gently over the rotating disc along the same axis. The energy dissipated so that both the discs continue to rotate together without slipping is _____ J.



Ans. (250)

Sol. $\vec{L}_i = I\omega_i = \frac{MR^2}{2}.\omega = 100\ kgm^2/s$

$$E_i = \frac{1}{2}.\frac{MR^2}{2}.\omega^2 = 500\,\mathbf{J}$$

$$\vec{L}_i = \vec{L}_f \ \Rightarrow 100 = 2I\omega_f$$

ω_f = 5 rad/sec

$$E_f = 2\times\frac{1}{2}.\frac{5(2)^2}{2}.(5)^2 = 250\,J$$

$$\Delta E = 250\,J$$

59. In a closed organ pipe, the frequency of fundamental note is 30 Hz. A certain amount of water is now poured in the organ pipe so that the fundamental frequency is increased to 110 Hz. If the organ pipe has a cross-sectional area of 2 cm^2, the amount of water poured in the organ tube is _______ g. (Take speed of sound in air is 330 m/s)

Ans. (400)

Sol. $\frac{V}{4\ell_1} = 30 \Rightarrow \ell_1 = \frac{11}{4}m$

$\frac{V}{4\ell_2} = 110 \Rightarrow \ell_2 = \frac{3}{4}m$

$\Delta\ell = 2m,$

Change in volume = $A\Delta\ell = 400\,cm^3$

M = 400 g ; $\left(\because \rho = 1g/cm^3\right)$

60. A ceiling fan having 3 blades of length 80 cm each is rotating with an angular velocity of 1200 rpm. The magnetic field of earth in that region is 0.5 G and angle of dip is 30º. The emf induced across the blades is $N\pi \times 10^{-5} V$. The value of N is _____ .

Ans. (32)

Sol. $B_v = B\sin 30 = \frac{1}{4} \times 10^{-4}$

$\omega = 2\pi \times f = \frac{2\pi}{60} \times 1200 \;\; rad/s$

$\varepsilon = \frac{1}{2} B_V \omega \ell^2$

$= 32\pi \times 10^{-5}$ V

SECTION-A

61. Given below are two statements:

Statement-I: The gas liberated on warming a salt with dil H_2SO_4, turns a piece of paper dipped in lead acetate into black, it is a confirmatory test for sulphide ion.

Statement-II: In statement-I the colour of paper turns black because of formation of lead sulphite.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement-I and Statement-II are false

(2) Statement-I is false but Statement-II is true

(3) Statement-I is true but Statement-II is false

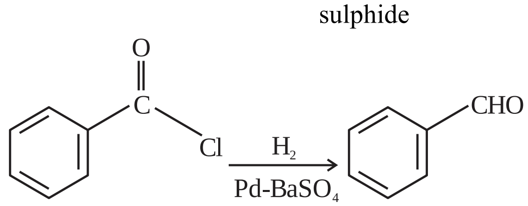
(4) Both Statement-I and Statement-II are true.

Ans. (3)

Sol. $Na_2S + H_2SO_4 \rightarrow Na_2SO_4 + H_2S$

$(CH_3COO)_2Pb + H_2S \rightarrow \underset{\text{Black lead sulphide}}{PbS} + 2CH_3COOH$

62.



This reduction reaction is known as:

(1) Rosenmund reduction

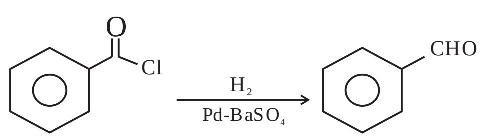
(2) Wolff-Kishner reduction

(3) Stephen reduction

(4) Etard reduction

Ans. (1)

Sol.



It is known as rosenmund reduction that is the partial reduction of acid chloride to aldehyde

63. Sugar which does not give reddish brown precipitate with Fehling's reagent is:

(1) Sucrose (2) Lactose

(3) Glucose (4) Maltose

Ans. (1)

Sol. Sucrose do not contain hemiacetal group.

Hence it does not give test with Fehling solution.

While all other give positive test with Fehling solution

64. Given below are the two statements: one is labeled as Assertion (A) and the other is labeled as Reason (R).

Assertion (A): There is a considerable increase in covalent radius from N to P. However from As to Bi only a small increase in covalent radius is observed.

Reason (R): covalent and ionic radii in a particular oxidation state increases down the group.

In the light of the above statement, choose the most appropriate answer from the options given below:

(1) (A) is false but (R) is true

(2) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(3) (A) is true but (R) is false

(4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Ans. (2)

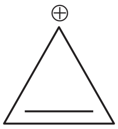
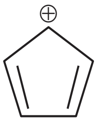
Sol. According to NCERT,

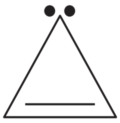
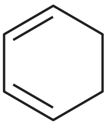
Statement-I : Factual data,

Statement-II is true.

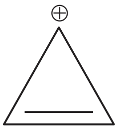
But correct explanation is presence of completely filled d and f-orbitals of heavier members

65. Which of the following molecule/species is most stable?

(1)    (2) 

(3)    (4) 

Ans. **(1)**

Sol.  it is aromatic species

66. Diamagnetic Lanthanoid ions are:

(1) Nd^{3+} and Eu^{3+} (2) La^{3+} and Ce^{4+}

(3) Nd^{3+} and Ce^{4+} (4) Lu^{3+} and Eu^{3+}

Ans. **(2)**

Sol. Ce : [Xe] $4f^1 5d^1 6s^2$; Ce^{4+} diamagnetic

La : [Xe] $4f^0 5d^1 6s^2$; La^{3+} diamagnetic

67. Aluminium chloride in acidified aqueous solution forms an ion having geometry

(1) Octahedral

(2) Square Planar

(3) Tetrahedral

(4) Trigonal bipyramidal

Ans. **(1)**

Sol. $AlCl_3$ in acidified aqueous solution forms octahedral geometry $[Al(H_2O)_6]^{3+}$

68. Given below are two statements:

Statement-I: The orbitals having same energy are called as degenerate orbitals.

Statement-II: In hydrogen atom, 3p and 3d orbitals are not degenerate orbitals.

In the light of the above statements, choose the **most appropriate** answer from the options given

(1) Statement-I is true but Statement-II is false

(2) Both Statement-I and Statement-II are true.

(3) Both Statement-I and Statement-II are false

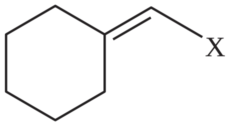
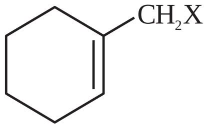
(4) Statement-I is false but Statement-II is true

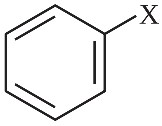
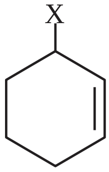
Ans. **(1)**

Sol. For single electron species the energy depends upon principal quantum number 'n' only. So, statement II is false.

Statement I is correct definition of degenerate orbitals.

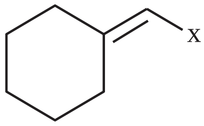
69. Example of vinylic halide is

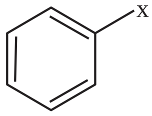
(1)    (2) 

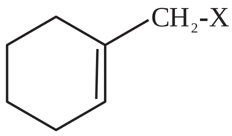
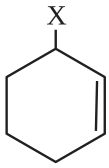
(3)    (4) 

Ans. **(1)**

Sol. Vinyl carbon is sp^2 hybridized aliphatic carbon

 is vinyl halide

While  is aryl halide

and   are allyl halide

70. Structure of 4-Methylpent-2-enal is

(1) $H_2C=C(H)-C(H)(CH_3)-CH_2-C(=O)-H$

(2) $CH_3-CH_2-C(CH_3)=CH-C(=O)-H$

(3) $CH_3-CH_2-CH=C(CH_3)-C(=O)-H$

(4) $CH_3-CH(CH_3)-CH=CH-C(=O)-H$

Ans. **(4)**

Sol. $\overset{5}{C}H_3 - \overset{4}{C}H(CH_3) - \underset{3}{C}H = \underset{2}{C}H - \underset{1}{C}(=O) - H$

4-Methylpent-2-enal

71. Match List-I with List-II

List-I	**List-II**
Molecule	**Shape**
(A) BrF_5	(I) T-shape
(B) H_2O	(II) See saw
(C) ClF_3	(III) Bent
(D) SF_4	(IV) Square pyramidal

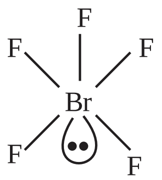
(1) (A)-I, (B)-II, (C)-IV, (D)-III

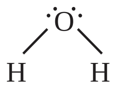
(2) (A) –II, (B)-I, (C)-III, (D)-IV

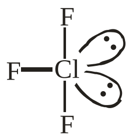
(3) (A)-III, (B)-IV, (C)-I, (D)-II

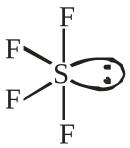
(4) (A)-IV, (B)-III, (C)-I, (D)-II

Ans. (4)

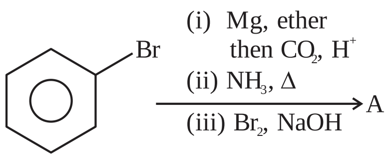
Sol. BrF_5  Square pyramidal

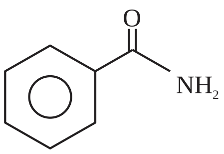
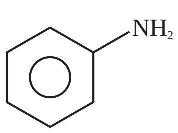
H_2O  Bent

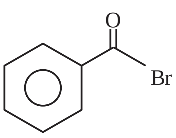
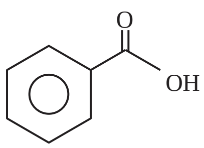
ClF_3  T-shape

SF_4,  See-saw

72. The final product A, formed in the following multistep reaction sequence is:

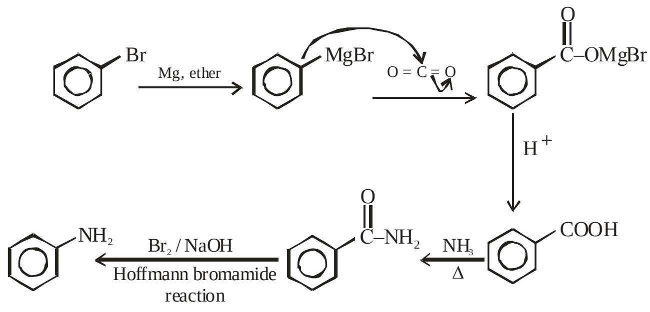


(1)  (2) 

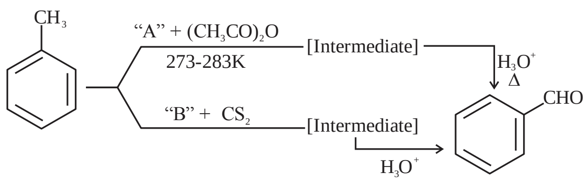
(3)  (4) 

Ans. (2)

Sol.



73. In the given reactions identify the reagent A and reagent B



(1) A-CrO_3 B-CrO_3

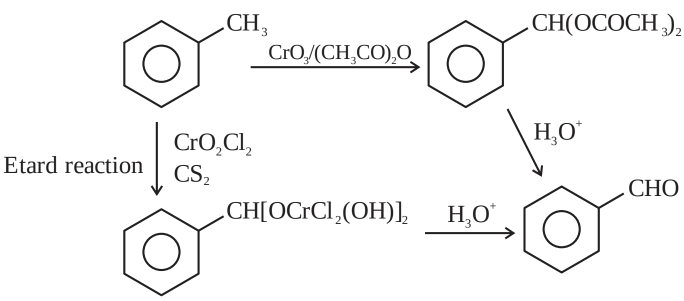
(2) A-CrO_3 B-CrO_2Cl_2

(3) A-CrO_2Cl_2 B-CrO_2Cl_2

(4) A-CrO_2Cl_2 B-CrO_3

Ans. (2)

Sol.



74. Given below are two statement one is labeled as **Assertion (A)** and the other is labeled as **Reason (R).**

Assertion (A): $CH_2 = CH - CH_2 - Cl$ is an example of allyl halide

Reason (R): Allyl halides are the compounds in which the halogen atom is attached to sp^2 hybridised carbon atom.

In the light of the two above statements, choose the **most appropriate** answer from the options given below:

(1) **(A)** is true but **(R)** is false

(2) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

(3) **(A)** is false but **(R)** is true

(4) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

Ans. (1)

Sol. $CH_2 = CH - CH_2 - Cl$

$\uparrow$

It is allyl carbon and sp^3 hybridized

75. What happens to freezing point of benzene when small quantity of napthalene is added to benzene?

(1) Increases

(2) Remains unchanged

(3) First decreases and then increases

(4) Decreases

Ans. (4)

Sol. On addition of naphthalene to benzene there is depression in freezing point of benzene.

76. Match List-I with List-II

List-I	**List-II**
Species	**Electronic distribution**
(A) Cr^{+2}	(I) $3d^8$
(B) Mn^+	(II) $3d^3 4s^1$
(C) Ni^{+2}	(III) $3d^4$
(D) V^+	(IV) $3d^5 4s^1$

Choose the correct answer from the options given below:

(1) (A)-I, (B)-II, (C)-III, (D)-IV

(2) (A)-III, (B) – IV, (C) – I, (D)-II

(3) (A)-IV, (B)-III, (C)-I, (D)-II

(4) (A)-II, (B)-I, (C)-IV, (D)-III

Ans. (2)

Sol. $_{24}Cr \rightarrow [Ar]\ 3d^5 4s^1;\ Cr^{2+} \rightarrow [Ar]\ 3d^4$

$_{25}Mn \rightarrow [Ar]\ 3d^5 4s^2;\ Mn^+ \rightarrow [Ar]\ 3d^5 4s^1$

$_{28}Ni \rightarrow [Ar]\ 3d^8 4s^2;\ Ni^{2+} \rightarrow [Ar]\ 3d^8$

$_{23}V \rightarrow [Ar]\ 3d^3 4s^2;\ V^+ \rightarrow [Ar]\ 3d^3 4s^1$

77. Compound A formed in the following reaction reacts with B gives the product C. Find out A and B.

$$CH_3 - C \equiv CH + Na \rightarrow A \xrightarrow{B} \underset{(C)}{CH_3 - C \equiv C - CH_2 - \underset{\displaystyle CH_3}{\underset{|}{CH_2}}} + NaBr$$

(1) $A = CH_3 - C \equiv \bar{C}\overset{+}{Na}$, $B = CH_3 - CH_2 - CH_2 - Br$

(2) $A = CH_3 - CH = CH_2$, $B = CH_3 - CH_2 - CH_2 - Br$

(3) $A = CH_3 - CH_2 - CH_3$, $B = CH_3 - C \equiv CH$

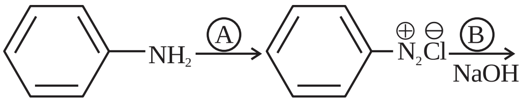
(4) $A = CH_3 - C \equiv \bar{C}\overset{+}{Na}$, $B = CH_3 - CH_2 - CH_3$

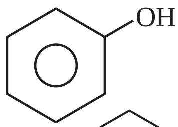
Ans. (1)

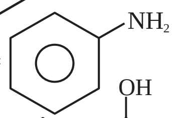
Sol.

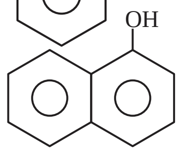
$$CH_3 - C \equiv CH \xrightarrow{Na} CH_3 - C \equiv C^- Na^+ \xrightarrow{CH_3CH_2CH_2 - Br} NaBr + CH_3 - C \equiv C - CH_2CH_2CH_3$$

78. Following is a confirmatory test for aromatic primary amines. Identify reagent (A) and (B)

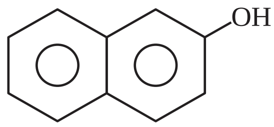


(1) $A = HNO_3/H_2SO_4$; B = 

(2) $A = NaNO_2 + HCl,\ 0 - 5°C$; B = 

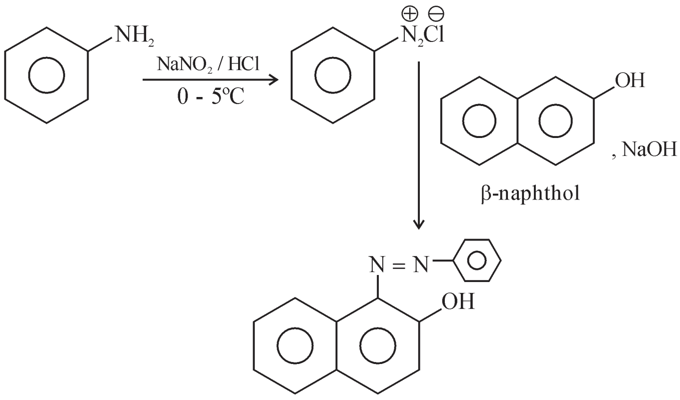
(3) $A = NaNO_2 + HCl,\ 0 - 5°C$; B = 

(4) $A = NaNO_2 + HCl,\ 0 - 5°C$;

B = 

Ans. (4)

Sol.



Scarlet red dye

79. The Lassiagne's extract is boiled with dil HNO_3 before testing for halogens because,

(1) AgCN is soluble in HNO_3

(2) Silver halides are soluble in HNO_3

(3) Ag_2S is soluble in HNO_3

(4) Na_2S and NaCN are decomposed by HNO_3

Ans. (4)

Sol. If nitrogen or sulphur is also present in the compound, the sodium fusion extract is first boiled with concentrated nitric acid to decompose cyanide or sulphide of sodium during Lassaigne's test

80. Choose the correct Statements from the following:

(A) Ethane-1 2-diamine is a chelating ligand.

(B) Metallic aluminium is produced by elecrtrolysis of aluminium oxide in presence of cryolite.

(C) Cyanide ion is used as ligand for leaching of silver.

(D) Phosphine act as a ligand in Wilkinson catalyst.

(E) The stability constants of Ca^{2+} and Mg^{2+} are similar with EDTA complexes.

Choose the correct answer from the options given below:

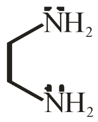
(1) (B), (C), (E) only

(2) (C), (D), (E) only

(3) (A), (B), (C) only

(4) (A), (D), (E) only

Ans. (3)

Sol. 

Bidentate, chelating

Based on Hall-Heroults process

$[Rh(PPh_3)_3Cl]$ Wilkinson's catalyst

$Ag_2S + NaCN \xrightleftharpoons{Air} Na[Ag(CN)_2] + Na_2S$

Ca^{++} ion forms more stable complex with EDTA

SECTION-B

81. The rate of first order reaction is 0.04 mol L^{-1} s^{-1} at 10 minutes and 0.03 mol L^{-1} s^{-1} at 20 minutes after initiation. Half life of the reaction is _______ minutes. (Given log2=0.3010, log3=0.4771)

Ans. (24)

Sol. $0.04 = k[A]_0 e^{-k \times 10 \times 60}$ ...(1)

$0.03 = k[A]_0 e^{-k \times 20 \times 60}$ ...(2)

(1)/(2)

$$\frac{4}{3} = e^{600k(2-1)}$$

$$\frac{4}{3} = e^{600k}$$

$$\ln\frac{4}{3} = 600k$$

$$\ln\frac{4}{3} = 600 \times \frac{\ln 2}{t_{1/2}}$$

$$t_{1/2} = 600\frac{\ln 2}{\ln\frac{4}{3}}\text{ sec}$$

$$t_{1/2} = 600 \times \frac{\log 2}{\log 4 - \log 3}\text{ sec.} = 10 \times \frac{0.3010}{0.6020 - 0.477}\text{ min}$$

$t_{1/2} = 24.08$ min

Ans. 24

82. The pH at which $Mg(OH)_2$ $[K_{sp} = 1 \times 10^{-11}]$ begins to precipitate from a solution containing 0.10 M Mg^{2+} ions is ______

Ans. (09)

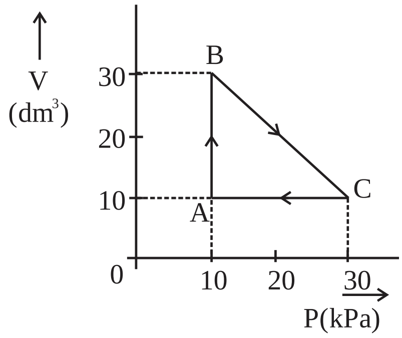
Sol. Precipitation when $Q_{sp} = K_{sp}$

$[Mg^{2+}][OH^-]^2 = 10^{-11}$

$0.1 \times [OH^-]^2 = 10^{-11} \Rightarrow [OH^-] = 10^{-5}$

$\Rightarrow$ pOH = 5 $\Rightarrow$ pH = 9

83.



An ideal gas undergoes a cyclic transformation starting from the point A and coming back to the same point by tracing the path A → B → C → A as shown in the diagram. The total work done in the process is _____ J.

Ans. **(200)**

Sol. Work done is given by area enclosed in the P vs V cyclic graph or V vs P cyclic graph.

Sign of work is positive for clockwise cyclic process for V vs P graph.

$W = \frac{1}{2} \times (30 - 10) \times (30 - 10) = 200\ kPa - dm^3$

$= 200 \times 1000\ Pa - L = 2\ L\text{-bar} = 200\ J$

84. if IUPAC name of an element is "Unununnium" then the element belongs to nth group of periodic table. The value of n is______

Ans. **(11)**

Sol. 111 belongs to 11^{th} group

85. The total number of molecular orbitals formed from 2s and 2p atomic orbitals of a diatomic molecule

Ans. **(08)**

Sol. Two molecular orbitals σ 2s and σ*2s.

Six molecular orbitals $\sigma\ 2p_z$ and σ^*2p_z.

$\pi 2p_x$, $\pi 2p_y$ and π^*2p_x, π^*2p_y

86. On a thin layer chromatographic plate, an organic compound moved by 3.5 cm, while the solvent moved by 5 cm. The retardation factor of the organic compound is _____ $\times 10^{-1}$

Ans. **(07)**

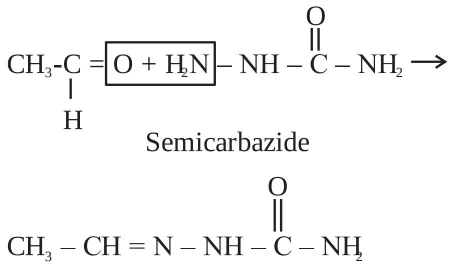
Sol. Retardation factor = $\frac{\text{Distance travelled by sample/organic compound}}{\text{Distance travelled by solvent}}$

$= \frac{3.5}{5} = 7 \times 10^{-1}$

87. The compound formed by the reaction of ethanal with semicarbazide contains _____number of nitrogen atoms.

Ans. **(03)**

Sol.



88. 0.05 cm thick coating of silver is deposited on a plate of 0.05 m^2 area. The number of silver atoms deposited on plate are _____ $\times 10^{23}$. (At mass Ag = 108, d = 7.9 g cm^{-3})

Ans. **(11)**

Sol. Volume of silver coating = $0.05 \times 0.05 \times 10000 = 25\ cm^3$

Mass of silver deposited = 25×7.9 g

Moles of silver atoms = $\frac{25 \times 7.9}{108}$

Number of silver atoms = $\frac{25 \times 7.9}{108} \times 6.023 \times 10^{23}$

$= 11.01 \times 10^{23}$

Ans. 11

89. $2MnO_4^- + bI^- + cH_2O \rightarrow xI_2 + yMnO_2 + zOH^-$

If the above equation is balanced with integer coefficients, the value of z is _______

Ans. (08)

Sol. **Reduction Half** / **Oxidation Half**

$2MnO_4^- \rightarrow 2MnO_2$; $2I^- \rightarrow I_2 + 2e^-$

$2MnO_4^- + 4H_2O + 6e^- \rightarrow 2MnO_2 + 8OH^-$; $6I^- \rightarrow 3I_2 + 6e^-$

Adding oxidation half and reduction half, net reaction is

$$2MnO_4^- + 6I^- + 4H_2O \rightarrow 3I_2 + 2MnO_2 + 8OH^-$$

$\Rightarrow z = 8$

$\Rightarrow$ Ans 8

90. The mass of sodium acetate (CH_3COONa) required to prepare 250 mL of 0.35 M aqueous solution is ______ g. (Molar mass of CH_3COONa is 82.02 g mol^{-1})

Ans. (7)

Sol. Moles = Molarity × Volume in litres

= 0.35 × 0.25

Mass = moles × molar mass

= 0.35 × 0.25 × 82.02 = 7.18 g

Ans. 7